Scand. J. of Economics 97(4), 495–526, 1995

A Theory of the Welfare State

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Abstract

The welfare state can be seen as an insurance device that makes lifetime careers safer, increases risk taking and suffers from moral hazard effects. Adopting this view, the paper studies the trade-off between average income and inequality, evaluating redistributive equilibria from an allocative point of view. It examines the problem of optimal redistributive taxation with tax-induced risk taking and shows that constant returns to risk taking are likely to imply a paradox where more redistribution results in more post-tax inequality. In general, optimal taxation will imply either that the redistribution paradox is present or that the economy operates at a point of its efficiency frontier where more inequality implies a lower average income.

I. Redistribution and Insurance

While this may be the time to turn the welfare state around, it is also the time to warn against throwing the baby out with the bathwater. Economists have learned so much about the Laffer curve, Leviathan, and a myriad of disincentive effects brought about by government intervention that they have lost sight of the allocative advantages of the welfare state.

From an allocative point of view, the main advantage of the welfare state is the insurance or risk reducing function of redistributive taxation. To finance commonly accessible public goods and public transfers, governments take more taxes from the rich than from the poor, thus reducing the variance in real lifetime incomes. To the extent that this variance is not predictable when people are born, this activity can be regarded as welfare increasing insurance. Every insurance contract involves a redistribution of resources from the lucky to the unlucky, and most of the redistributive

*This paper is part of the NBER program on public economics and the CEPR project "Monitoring European Integration". The author wishes to thank the referees and the participants of *SJE's* conference on "The Future of the Welfare State" for useful comments. Research assistance by Marcel Thum is also gratefully acknowledged.

measures of the state can be interpreted as insurance if the time span between judging and taking these measures is sufficiently long. Redistributive taxation and insurance are two sides of the same coin.

It has been argued that the insurance function of the government budget can be privately provided and that redistributive taxation might simply crowd out private insurance; see Kaplow (1991, 1992) and Konrad (1991). This argument certainly has theoretical appeal for a number of specific risks. However, it does not seem applicable to the typical lifetime income risk. It is difficult to imagine endowing private agencies with the extensive monitoring and enforcement rights which the government needs in order to administer an income tax, and in the absence of such rights, moral hazard and adverse selection problems render a broad-based private solution impossible.[1] The insurance provided by the public tax and transfer system is an insurance against the randomness in career opportunities and in nature's lottery draw of innate abilities. Organizing this insurance privately would require signing a contract with a lifelong commitment at the time of birth of an individual; it would approximate bondage, a system long overcome by the course of history. In addition, as pointed out by Christiansen (1990), government insurance may well be cheaper than private insurance given that a system of fiscal taxation is considered inevitable. The marginal cost of making the existing tax system redistributive will, in all likelihood, be lower than the total cost of introducing private income insurance *ab ovo*. Regardless of which of these reasons dominated, the historical growth of the welfare state can, in part, be seen as a response to the inability of the private insurance system to offer the better solution.[2]

While the production of safety is an important function of the welfare state, the Domar–Musgrave effect of increased risk taking may be even more important. Protected by the welfare state, people engage in risky and profitable activities which they otherwise would not have dared to undertake. Risky occupations might not be chosen without the protection of the welfare state, and it would be difficult to find entrepreneurs willing to supervise risky investment if debtor's prison were all that society provided in the case of failure. Perhaps the most important function of the social welfare net is that it makes people jump over the dangerous chasms which would otherwise have put a halt to their economic endeavors.

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[1] For an explicit adverse selection model where a positive role is left for insurance through the tax system, see Konrad (1992, pp. 126–8).

[2] An enlightening discussion of further reasons for the government's superior ability to absorb income risk is provided by Gordon (1985) and Gordon and Varian (1988). These reasons include intergenerational diversification in the absence of an operative bequest motive as well as diversification in the form of changing the supply of public goods.

It may, in fact, make them too eager to jump. Protected by the welfare state, people may neglect to take necessary care, may take too much risk, and end up in a worse situation than without such protection. This is the moral hazard problem that an overwhelming majority of policy advisors seems to fear. The paper offers a simple model that makes it possible to analyse the interaction between redistributive taxation and risk taking, distinguishing sharply between a desirable increase in risk taking and an overshooting in risk taking due to moral hazard effects.

The effect on risk taking has important repercussions for the observable degree of inequality in the economy, for, if a given set of people choose more risk *ex ante*, they will typically be more unequal *ex post*. Risk averse societies may exhibit relatively little inequality, and the more redistribution there is, the larger the pre-tax inequality tolerated may be. As suggested by Harsanyi (1953, 1955), Rawls (1971) and others, the social welfare function for evaluating the income distribution is taken to be identical with a representative individual's utility function for risk evaluations. However, unlike the argument brought forward by these authors, in the model, people really are behind the veil of ignorance when they make their decisions and evaluate the resulting income distribution. Their amount of risk taking *ex ante* determines their degree of inequality *ex post*.

The main focus here is on the policy trade-off between income equality and average income. It is not on the trade-off between equity and efficiency, because equity is an aspect of efficiency. Will redistributive taxation induce too much or too little risk taking? How does it compare with ideal insurance? Will the pie shrink when it is more evenly distributed? Will more redistribution result in less inequality? What are the characteristics of an optimal redistributive tax system that balances the marginal impacts on the size of the pie and the equality in the slices distributed? These are among the questions addressed in this paper.

While little is known about the issue, there are many important studies on the role of taxation under uncertainty. These include the literature on risk taking and taxation in the context of asset choice, savings or occupational decisions, e.g. Ahsan (1974, 1976), Allingham (1972), Atkinson and Stiglitz (1980, Ch. 4), Bamberg and Richter (1984), Domar and Musgrave (1944), Kanbur (1979), Sandmo (1977) and Sinn (1981), as well as the welfare theoretic literature studying optimal redistributive taxation in the case of income risks, e.g. Diamond, Helms and Mirrlees (1980), Eaton and Rosen (1980), Varian (1980) and Rochet (1991). This paper is an attempt to integrate some of the existing ideas by analyzing the problem of optimal redistributive taxation in the context of tax-induced risk taking. The first literature mentioned has not considered the problem of optimal taxation, and the other has not been concerned with the issue of risk taking. Combining the two issues may offer new insight into the nature of the welfare state and help derive new propositions about the trade-off between income and equality.

In considering the modern literature, it should not be forgotten that the paper's basic themes were first discussed in Friedman's (1953) "Choice, Chance and the Personal Distribution of Income" and Buchanan and Tullock's (1962) *Calculus of Consent*, Chapter 13. The analysis can be understood as an attempt to formalize, apply and develop these path-breaking approaches.

A technical feature distinguishing the present model from the existing literature and allowing new questions to be asked is the location and scale parameter methodology developed by Meyer (1987) and Sinn (1983, 1989) which makes it possible to represent the individual choice problem and the resulting income distribution in a (μ, σ) framework without imposing the usual restrictions on preferences and technologies. Despite the assumption of expected utility maximization, this methodology is based neither on quadratic utility nor on normal distributions. The use of an additional result concerning the required marginal compensation for risk taking reported in Sinn (1990) makes it possible to find strong implications of redistributive taxation while avoiding the familiar ambiguities in the relationship between taxation and risk taking pointed out by Feldstein (1969) and Stiglitz (1969) for the case of fiscal taxation.

II. The Model

A very simple model that is able to incorporate the issues discussed is the following. There is a large number of identical individuals, each facing the same choice problem under uncertainty. With stochastically independent income risks and identical choices, each person's probability distribution of income converts to the economy's frequency distribution of realized incomes. If, say, a single person's probability of having a lifetime income of between \$500,000 and \$510,000 is 1 per cent, then the law of large numbers will ensure that 1 per cent of the population will have an income in this range. Risk and expected income *ex ante* will turn out as inequality and average income *ex post*.

To reduce the dimensionality of risk, a broad-based definition of income including market income, non-market income (or leisure), public goods and public transfers is used. The risk occurs in the form of an uninsurable lifetime random income loss $L \geq 0$ whose magnitude depends on the random state of nature θ and the cost of self-insurance effort e in terms of foregone market and non-market resources. The variable θ may, for example, reflect the risk in unknown innate abilities or uncontrollable external events, and e may stand for working time or investment in physical and human capital limiting the risk of not reaching one's income goals. Let m and n be the maximum values of market and non-market income attainable if the individual makes no effort and the loss nevertheless happens to be zero, p be the value of transfers (monetary transfers and public goods) received, and T be the individual's tax liability which, among other things, also depends on θ and e. Then the individual's (post-tax) income is

$$Y = m + n - L(e, \theta) - e - T(e, \theta) + p. \tag{1}$$

Effort is chosen before nature has revealed θ. An increase in effort e reduces the size of the income loss for all states of the world, albeit with diminishing marginal returns. It is assumed that[3]

$$L(e, \theta) = \lambda(e)\,\theta, \qquad \theta \geq 0, \qquad \lambda > 0, \tag{2}$$

$$\lambda' < 0, \qquad \lambda'' \geq 0, \qquad \lambda'(0) = -\infty,$$

where λ is a twice continuously differentiable function reflecting the efficacy of *self-insurance* — to use a term first introduced by Ehrlich and Becker (1972).

There is a linear tax on market income. Let α be the fraction of self-insurance efforts consisting of foregone market income and $1 - \alpha$ the fraction consisting of foregone non-market income (or leisure). Then

$$T(e, \theta) = \tau[m - L(e, \theta) - \alpha e] \tag{3}$$

where τ is the tax rate. Note that, despite the linearity of the tax, the tax system is redistributive because the public transfer p is independent of the state of nature.[4] Lucky individuals are net payers and unlucky net recipients of public funds. While α is treated as an exogenous parameter throughout this paper, τ is endogenously determined in a social optimization problem in Section VI.

To balance the government budget, the public transfer is chosen so as to make it equal to the average tax liability:[5]

$$p = E[T(e, \theta)]. \tag{4}$$

[3] Note that this formulation differs significantly from that of Varian (1980) where the individual is assumed to be unable to affect his income risk through his own actions. In Varian's model, the (μ, σ) trade-off specified below would have to be represented by a vertical straight line.

[4] The formal structure of the redistribution mechanism is similar to the progressive linear tax used by Ahsan (1974, 1976) for a portfolio selection problem with fiscal taxation.

[5] Alternatively, it could have been assumed that $p = \sum_{j=1}^{x} T(e_j, \theta_j)/x$ where x is the number of individuals in the economy. Because of the assumption of identical choices and stochastic independence of the θ_j, $j = 1, \ldots, x$, the transfer specified this way converges stochastically to $E[T(e, \theta)]$ as x goes to infinity.

It is assumed that the government can observe m, n and the individual realization of L, and that it learns the tax deduction αe legally claimed by each individual according to the specifications given in the tax law. The government has some statistical information on α which makes it possible to infer the underlying effort level chosen by the average taxpayer, but it may be unable to observe the individual effort level e or be unwilling to make it fully tax deductible.[6] Similarly, the government possesses the statistical information necessary for choosing the transfer p so as to satisfy its budget constraint (4), but it is unable to tailor each individual's transfer p to this individual's expected tax liability. Equation (4) holds in equilibrium without implying that the individual is able to change p through his own actions.

The formulation includes the extreme cases $\alpha = 0$ and $\alpha = 1$. In the case $\alpha = 0$, the opportunity cost of effort occurs exclusively in the form of foregone non-market income, and non-market income is unobservable and untaxed. This case can be interpreted in terms of the familiar labor-leisure distortion if leisure is, in fact, an activity producing non-market income and if the tax is imposed on labor income alone. The tax system discourages the self-insurance effort because this effort cannot be deducted from the tax base. In the case $\alpha = 1$, and only in this case, individual effort is fully observable. It occurs exclusively in the form of foregone market resources and will enjoy full tax deductibility. One may think in particular of pecuniary investment outlays or business expenses that are fully tax deductible. In an intertemporal context, an ideal cash flow tax would be an exact example for the case $\alpha = 1$ because it allows an immediate write-off of investment expenses.[7] A capital income tax with annual economic depreciation allowances would instead be equivalent to $0 < \alpha < 1$, because the present value of depreciation allowances falls short of the investment. It will be shown below that whenever $\alpha < 1$, there is a moral hazard effect in terms of reduced effort strong enough to imply an optimal tax rate less than unity.[8] Only in the theoretical case $\alpha = 1$ would it be optimal to fully develop the welfare state.

It is admissible to assume that there is a perfect private insurance market in the background that has already absorbed some of the risks the individual would otherwise have to bear. It simply had to be assumed that m and n are incomes net of the respective insurance premia where m is an

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[6] The analysis abstracts from the problem of imperfect observability of losses as may be the case with health insurance; for a discussion of such issues see Diamond and Mirrlees (1978).
[7] The variables of the model would then have to be interpreted in terms of present values.
[8] Note that the case $\alpha < 1$ can also be interpreted as describing a situation where all self-insurance occurs in the form of foregone market resources, but where not all of these resources are tax deductible.

income net of tax deductible, and n an income net of non-deductible, premia.[9] Recall that L is the uninsurable risk in one's lifetime career which may largely result from the randomness in nature's draw of innate abilities.

The income distribution in the economy described is specified once the government has chosen τ and the individuals have chosen e. For the planned analysis of income distributions, it is convenient to describe this distribution in terms of its mean μ (the average income) and its standard deviation σ. It follows from (1)–(4) that

$$\mu = m + n - \lambda(e)E(\theta) - e \tag{5}$$

and

$$\sigma = (1-\tau)\lambda(e)R(\theta) \tag{6}$$

where $R(\cdot)$ is the standard deviation operator.[10] Equations (5) and (6) show that, with any given amount of self-insurance effort e, redistributive taxation will not affect the average income, μ, but will reduce the deviation from the average, σ. Seen from an *ex-ante* perspective, this is the insurance aspect of redistributive taxation. The important question of how redistributive taxation will in turn affect the amount of effort chosen is postponed to later sections.

Figure 1 depicts the combinations of μ and σ attainable with an appropriate choice of e and for two alternative values of the tax rate: $\tau = 0$ and $\tau > 0$, where σ is the post-tax and σ_G the pre-tax standard deviation of income.

The opportunity set of (μ, σ) combinations attainable with $\tau = 0$ will be called the "self-insurance line" and the set attainable with a given $\tau > 0$ will be called the "redistribution line". Geometrically, the redistribution line can be constructed by shifting all points on the self-insurance line horizontally to the left where the percentage reduction of the distance from the ordinate equals the tax rate. The movements of A, B and C towards A', B' and C' are examples of this shift. It is unclear at this stage which amount of self-insurance effort and which pair of points on the two lines the individual chooses. However, whatever his choice, all attainable post-tax income distributions that satisfy the government's budget constraint (4) are represented by points on the redistribution line.

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[9] See Rochet (1991) for a model that explicitly incorporates insurable and non-insurable risks where the redistributive tax system covers the latter.

[10] Throughout the paper E and R are used as expectation and standard deviation operators while μ and σ are the mean and standard deviation of post-redistribution income. Recall that

$R(X) = [E(X^2) - E^2(X)]^{1/2}$ and note that $E(a+bX) = a + bE(X)$ and $R(a+bX) = |b|R(X)$.

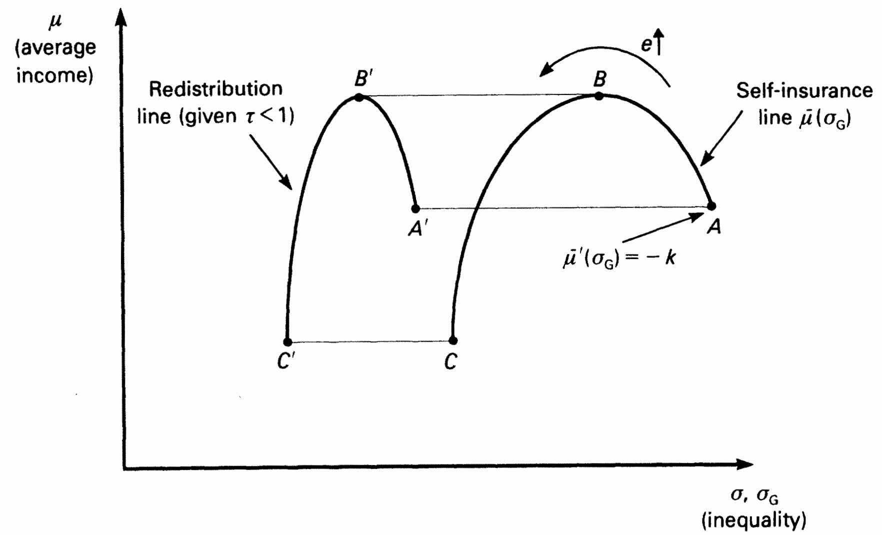

Fig. 1. The sets of feasible pre-tax and post-tax distributions of income.

The pre-tax standard deviation is given by

$$\sigma_G = \lambda(e)R(\theta). \tag{7}$$

Since $\lambda'(e) < 0$ implies that σ_G is a monotonically declining function of e, it is possible to treat σ_G as the choice variable of the individual. Accordingly (5) and (6) can be written as

$$\mu = \bar{\mu}(\sigma_G) \tag{8}$$

and

$$\sigma = (1-\tau)\sigma_G \tag{9}$$

where

$$\begin{aligned}\bar{\mu}(\sigma_G) &\equiv m + n - E(L) - e \\ &= m + n - \sigma_G k - \lambda^{-1}[\sigma_G/R(\theta)]\end{aligned} \tag{10}$$

is the function defining the self-insurance line with

$$\sigma_G k = E[L(e,\theta)] = \lambda(e)E(\theta) \tag{11}$$

and

$$k \equiv \frac{E(\theta)}{R(\theta)} = \text{const.} > 0. \tag{12}$$

It is easy to derive a boundary condition for the slope of the self-insurance line,[11]

$$\bar{\mu}'(\sigma_G) = -k \qquad \text{when } e = 0, \tag{13}$$

and to show that the line has a maximum where $\lambda'(e)E(\theta) = -1$ and is concave throughout:[12]

$$\bar{\mu}''(\sigma_G)\begin{Bmatrix} < \\ = \end{Bmatrix}0 \qquad \text{when } \lambda''(e)\begin{Bmatrix} > \\ = \end{Bmatrix}0. \tag{14}$$

To close the model, the representative agent's preference structure has to be specified. It is assumed that the agent is a globally and locally risk averse expected utility maximizer. Since the set of distributions implied by (1), (2) and (3) forms a linear class, any given von Neumann–Morgenstern function can be exactly represented in terms of (μ, σ) preferences without any loss of generality.[13] Neither quadratic utility nor normality in the distributions have to be assumed. As shown by Meyer (1987) and Sinn (1983, 1989), there exists a well-behaved utility function $U(\mu, \sigma)$ if the von Neumann–Morgenstern function is well behaved. Its properties can best be summarized by the properties of the function

$$i(\mu, \sigma) \equiv \left.\frac{\mathrm{d}\mu}{\mathrm{d}\sigma}\right|_U = -\frac{U_\sigma}{U_\mu} \tag{15}$$

which indicates the indifference-curve slope — required marginal compensation for risk — at a particular combination of μ and σ:

(a) $i(\mu, 0) = 0$ (enter ordinate perpendicularly)
(b) $i(\mu, \sigma) > 0$ for $\sigma > 0$ (upward bending)

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[11] Equation (13) follows from (5), (7) and the assumption $\lambda'(0) = -\infty$.

[12] It follows from (10) that $\bar{\mu}''(\sigma_G) = \lambda''(e)/[\lambda'^3(e)R^2(\theta)]$. Since $\lambda'' \geq 0$ and $\lambda' < 0$ the sign of this expression is zero or negative.

[13] To prove that the attainable distributions belong to the same linear class, it is necessary to show that the standardized distribution $Z = [Y - E(Y)]/R(Y)$ is independent of the model's choice variables and parameters e, τ and α. Inserting (2) and (3) into (1) gives

$$Z = \frac{m + n - \lambda\theta - e - \tau[m - \lambda\theta - \alpha e] + p - \{m + n - \lambda E(\theta) - e - \tau[m - \lambda E(\theta) - \alpha e] + p\}}{(1-\tau)\lambda R(\theta)}$$

or, after a few simplifications,

$$z = \frac{-\theta + E(\theta)}{R(\theta)}, \qquad \text{q.e.d.}$$

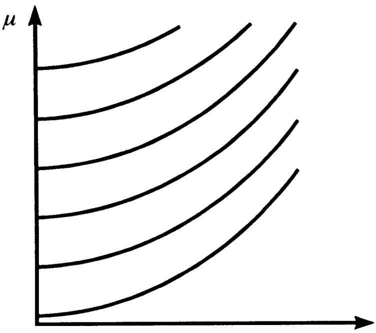

Fig. 2. Evaluating income distributions.

(c) $\left.\dfrac{\mathrm{d}i}{\mathrm{d}\sigma}\right|_U > 0$ (strictly convex)

(d) $i_\sigma > 0$ (slope increases with σ, given μ)[14]

(e) $i_\mu \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ for $\begin{Bmatrix} \text{increasing} \\ \text{constant} \\ \text{decreasing} \end{Bmatrix}$ absolute risk aversion (slope change with μ, given σ).

Figure 2 illustrates an example of the indifference-curve system for the case of constant absolute risk aversion. While the preference map of Figure 2 makes it possible to evaluate probability distributions, it allows an equally appropriate evaluation of the realized income distributions. Since people have identical risk preferences and since the probability distribution chosen translates into an identical frequency distribution of realized incomes, an unambiguous social welfare function is available.

III. *Laissez Faire* and the Social Optimum

Imposing the "indifference map" of Figure 2 on the "feasibility map" of Figure 1 gives two kinds of optima, illustrated by points T and Q' in Figure 3. Point T is the *laissez-faire* optimum without redistributive taxation and Q' is the optimum with redistribution at a given tax rate $\tau > 0$. Let T' and Q be the counterparts of these two points on the redistribution

[14] Condition (d) derives basic results of this paper. It has been proved under the condition that absolute risk aversion is decreasing, is constant, or does not increase faster than with the "fastest" quadratic utility function compatible with strictly positive marginal utility in the relevant range; see Sinn (1990). It is assumed that this extremely weak condition will hold.

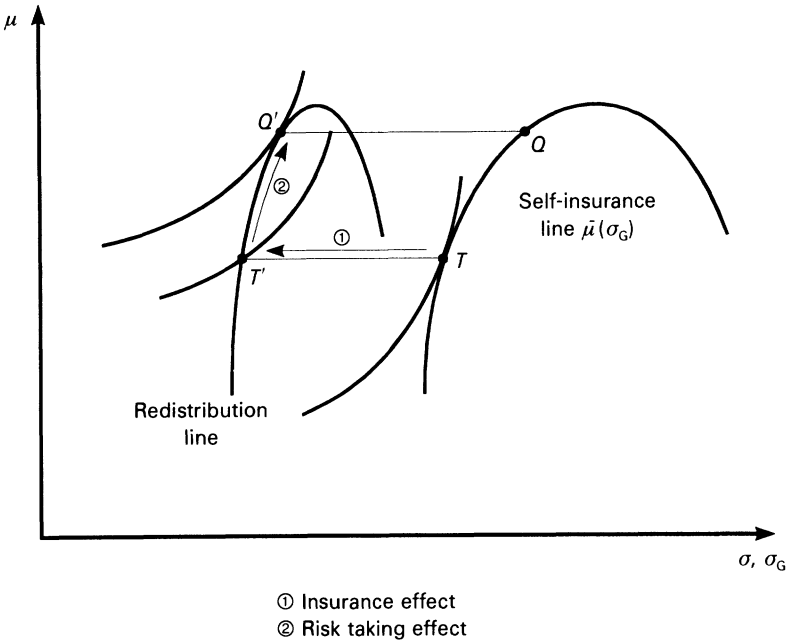

Fig. 3. The socially optimal degree of risk taking, given the tax rate.

line and the self-insurance line, respectively.[15] Formally, the two solutions follow from the problem

$$\max_{\sigma_G} U(\mu, \sigma) \qquad \text{s.t. } \mu = \bar{\mu}(\sigma_G), \quad \sigma = (1-\tau)\sigma_G \tag{16}$$

which implies the first-order condition

$$i[\bar{\mu}(\sigma_G), (1-\tau)\sigma_G] = \frac{\bar{\mu}'(\sigma_G)}{1-\tau}. \tag{17}$$

the l.h.s. of (17) is the indifference curve slope and the r.h.s. is the slope of the redistribution line. In general, (17) refers to a point like Q'; however, in the limiting case where $\tau = 0$ it also captures the *laissez-faire* solution T.

The solution illustrated in Figure 3 is a constrained Pareto optimum, defining the optimal level of self-insurance effort given the tax rate. It will not necessarily be reached by private actions, since the redistribution line

[15] Throughout the paper, points labelled with a prime are located on the redistribution line horizontally left of the respective points without a prime which are located on the self-insurance line. Points labelled by the same letter indicate the same self-insurance effort.

may not coincide with the opportunity set as perceived by the individual. It would, however, be attained in an ideal insurance market where individual actions can be monitored by the company and a fair premium is announced for each self-insurance strategy the individual may choose. It would also be attained if a strict equivalence principle of taxation could be met. The government would have to be able to monitor individual self-insurance activities and announce a separate value of the public transfer for every feasible action, obviously an unrealistic requirement.

Having made these reservations, two lessons can be learned from Figure 3.

Proposition 1. *Under laissez faire, or with ideal insurance, society operates at a point in its opportunity set where an increase in inequality would increase the average income.*

Proposition 2. *Redistributive taxation with individually tailored transfers creates two kinds of welfare gain. It increases welfare by increasing the equality of incomes, and it increases it even more when more risk is taken and some equality is sacrificed for a higher level of average income. The socially optimal level of pre-tax inequality is an increasing function of the tax rate.*

While Proposition 1 is obvious, Proposition 2 needs a

Proof: Assume that $0<\tau<1$ and let $r(\cdot)$, $i(\cdot)$, and $s(\cdot)$ denote the slopes of the redistribution line, the indifference curve and the self-insurance line at the respective points (in Figure 3) named in the brackets. By the definition of T, $s(T)=i(T)$ and, because of (8) and (9), $r(T')=r(T)/(1-\tau)>i(T)$. Property (d) of the indifference curve system ensures that $i(T)>i(T')$. Thus $r(T')>i(T')$. Together with the convexity of the indifference curves and the concavity of $\bar{\mu}$, this implies $\sigma(Q')>\sigma(T')$ and $\sigma_G(Q)>\sigma_G(T)$.[16] While this proves that taxation increases risk taking and pre-tax inequality in the large, the marginal effect of τ on the optimal level of σ_G, $\sigma_G(Q)$, follows from implicitly differentiating (17):

$$\frac{\mathrm{d}\sigma_G(Q)}{\mathrm{d}\tau}=\frac{i+i_\sigma\sigma_G(Q)(1-\tau)}{(1-\tau)\{i_\mu\bar{\mu}[\sigma_G(Q)]+i_\sigma(1-\tau)\}-\bar{\mu}''[\sigma_G(Q)]}>0. \tag{18}$$

The denominator of this expression is strictly positive if the second-order condition of problem (17) is satisfied. This is the case since the indif-

[16] The notation should be self-explanatory. For example $\sigma(T')$ is the post-tax standard deviation at point T' which is the counterpart of $\sigma_G(T)$, the pre-tax standard deviation. Note that $\sigma(T')=(1-\tau)\sigma_G(T)$.

ference curves are strictly concave and the redistribution line is convex. The numerator is strictly positive since all items occurring there are strictly positive. (Cf. property (d) of the indifference curve system.) **Q.e.d.**

Proposition 1 is the model's confirmation of the frequently expressed belief that the pie can grow when a more unequal distribution of its slices is tolerated. Risk aversion (or inequality aversion) requires a compromise between the goals of maximizing the size of the pie and minimizing the degree of inequality. It makes it wise to operate at a point on the efficiency frontier where a little more tolerance with regard to the latter makes it possible to come somewhat closer to the former.

Proposition 2 confirms the discussion in the introduction. Given that the government offers public insurance, the need for self-insurance is reduced. Redistributive taxation with individually tailored transfers increases the marginal post-tax return to risk taking (the slope of the redistribution line as compared to that of the self-insurance line) and lowers the marginal compensation for risk taking that the agent requires (the indifference curve slope). This makes it socially optimal to tolerate more risk and inequality in exchange for a higher level of average income. Under the protection of the welfare state, more can be dared.[17]

The risk taking effect of the welfare state may have far-reaching implications. In a broader context, risk can be seen as a factor of production, a necessary input for the economy without which a high level of productivity could not be achieved.[18] The factor "risk" is probably no less important than "waiting", the factor economists have familiarized themselves with under the name of capital. If the real rate of interest is a measure of the importance of waiting and if the unexplained remainder of the "return to capital" is in fact the reward for risk taking, then risk taking should be considered at least as responsible for economic prosperity as capital investment. The enhancement of risk taking may be the most important economic function the welfare state can perform.

IV. Redistributive Taxation and the Optimality of Individual Choice

While the preceding section demonstrated the potential for gains from redistributive taxation under rather unrealistic conditions, this section addresses the more interesting question of whether the exploitation of this

[17] Surprisingly, the benefits from increased risk taking have been largely ignored in the insurance literature. Often the insurance-induced increase in risk taking is confused with moral hazard resulting from a lack of observability of individual actions.

[18] See Pigou (1932, Appendix I, pp. 771–81), Sinn (1986) or Konrad (1992).

potential through individual choice can really be expected. The crucial assumption here is that the government transfer p is *not* tailored to the individual decision. The individual agent takes this transfer as exogenous to his own decisions, notwithstanding the fact that it will endogenously be determined in equilibrium through the government budget constraint, equation (4).

The individual opportunity set of decision alternatives is given by equation (1). Taking expectations, noting that $\bar{\mu}(\sigma_G) = m + n - E(L) - e$ from (10), and using (3) yields

$$\mu = \bar{\mu}(\sigma_G) - \tau\{m - E[L(e, \theta)] - ae\} + p. \tag{19}$$

After a few algebraic manipulations making use of (11), equation (19) can also be written as

$$\mu = \bar{\mu}(\sigma_G)(1 - a\tau) - \tau(1 - a)(m - k\sigma_G) + a\tau n + p. \tag{20}$$

The standard deviation as perceived by the individual follows from (1), (3) and (7):

$$\sigma = (1 - \tau)\sigma_G. \tag{21}$$

Since p was also non-stochastic in the social planning problem, this is the same as equation (9). Equations (20) and (21) imply an opportunity locus in (μ, σ) space that will be called the "individual opportunity line".

The agent's optimization problem is

$$\max_{\sigma_G} U(\mu, \sigma) \qquad \text{s.t.} \quad (20) \text{ and } (21). \tag{22}$$

Using (15), the first-order condition of this problem can be written as[19]

$$i(\mu, \sigma) = \bar{\mu}'(\sigma_G)\frac{1 - a\tau}{1 - \tau} + \frac{\tau}{1 - \tau}(1 - a)k. \tag{23}$$

The l.h.s. of equation (23) is the indifference curve slope, and the r.h.s. is the slope of the individual opportunity line.

A redistributive equilibrium is defined as a situation where the agent has chosen σ_G so as to maximize his utility and the government has chosen the public transfer p so as to satisfy its budget constraint (4). In equilibrium, therefore, (23) has to hold on the redistribution line (cf. Figures 1 and 3) which means that the indifference curve slope $i(\mu, \sigma)$ refers to a point where $\mu = \bar{\mu}(\sigma_G)$ and $\sigma = (1 - \tau)\sigma_G$.

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[19] The second-order condition is satisfied since the indifference curves are convex and (20) and (21) define a concave curve in (μ, σ) space representing the individual opportunity set as perceived by the agent.

A comparison with (17) reveals that the equilibrium satisfying (23) is not in general identical with the constrained Pareto optimum characterized by the pair (Q, Q') in Figure 3. The next three subsections analyze the differences.[20]

Deductible Effects

Consider first the case $\alpha = 1$, where, as explained, the cost of self-insurance occurs exclusively in the form of foregone market resources and will therefore enjoy full tax deductibility (cash flow tax). The implications of (23) for this case are summarized in

Proposition 3. *When self-insurance efforts are fully tax deductible (as with investment under a cash flow tax), redistributive taxation is welfare increasing. In addition to the direct gain from insurance there is a gain from increased risk taking. However, risk taking and the resulting increase in inequality are less than what would be socially optimal.*

Proof: If $\alpha = 1$, condition (23) becomes

$$i[\bar{\mu}(\sigma_G), (1-\tau)\sigma_G] = \bar{\mu}'(\sigma_G). \tag{24}$$

Assume that $\tau > 0$ and let $i(\cdot)$ and $s(\cdot)$ denote the slopes of the indifference curve and the self-insurance line at the respective points (from Figure 4) named in the brackets. Condition (24) defines a point V' on the redistribution line and its counterpart V horizontally to the right on the self-insurance line such that the indifference curve slope on the redistribution line equals the corresponding slope of the self-insurance line: $i(V') = s(V)$. From (17) it is known that $i(Q') = s(Q)/(1-\tau) > s(Q)$. On the other hand, property (d) of the indifference curve system and the definition of T imply that $i(T') < i(T) = s(T)$. Continuity implies that a solution exists between T' and Q' on the redistribution line; i.e., $\sigma(T') < \sigma(V') < \sigma(Q')$ and $\sigma_G(T) < \sigma_G(V) < \sigma_G(Q)$, **q.e.d.**

The intuition for the suboptimality of individual risk taking can best be gained by inspecting (19). Suppose the individual had chosen the socially otpimal level of σ_G and considers a small variation by changing his self-insurance effort. This variation will, in general, change his expected tax liability, $\tau\{m - E(L) - \alpha e\}$. If the public transfer p is changed accordingly so as to satisfy the government budget constraint (4), then the variation in σ_G implies no change in the expected net payment to the government, and, by assumption, expected utility stays constant. However, if p stays constant despite the change in the expected tax liability, expected utility will change. The individual will have an incentive to deviate from the social optimum in

[20] The existence of equilibrium is also proved in these subsections.

the direction where the expected tax liability declines and where he can expect to become a net recipient of public funds. Assuming an endogenous change in p would require collective rationality. It is when only individual rationality is available that p has to be taken as exogenous, because the agent knows that his taxes will contribute only a negligible fraction to the government budget and will therefore not be able to affect the volume of public transfers returned.

For the case $\alpha = 1$, this argument implies that the representative agent takes less risk and chooses a lower degree of inequality than is socially optimal, optimality being judged by his own preferences. The expected tax base is $\{m - E(L) - e\}$. Since it differs from the expected income $\bar{\mu}(\sigma_G)$ only by the non-market component of income, n, which is a constant, the expected tax liability can be reduced by lowering income and enjoying the advantage of lower risk.

Figure 4 illustrates this reasoning. The broken line through Q' is the individual opportunity line, given the level of public transfers p that would be paid if the agents chose the socially optimal level of self-insurance effort. The individual believes that he will be able to reach a higher indifference curve by moving to the left of Q'; i.e., by reducing σ_G. In fact, however, if everyone does so, the transfer will have to be reduced and the

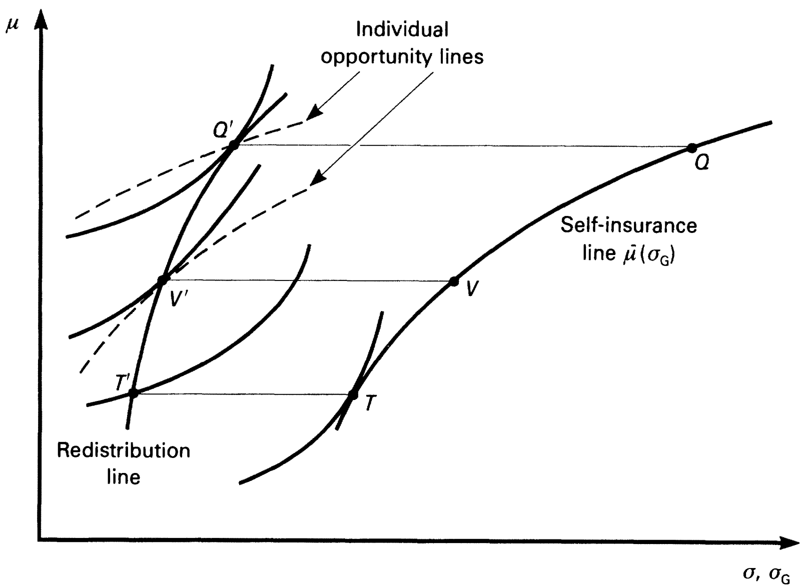

Fig. 4. Less than optimal inequality with full deductibility of self-insurance efforts (cash flow tax).

realized point in (μ, σ) space is pushed down, back to the redistribution line. The equilibrium is at a point such as V'. Here an indifference curve is tangent to an individual opportunity line, and the point of tangency is also on the redistribution line. The individual does not want to change his behavior, and the government budget is balanced.

It is important to note that, although the increase in risk taking is too small, there definitely is such an increase. Redistributive taxation without individually tailored transfers and with full deductibility of self-insurance efforts does not change the marginal post-tax return to risk taking (the slope of the individual opportunity line), but it lowers the required marginal compensation for risk taking (the slope of the indifference curve). This induces the individual to dare more in order to enjoy a higher level of expected income. There are no ambiguities of the kind Feldstein (1969) and Stiglitz (1969) pointed out for the case of fiscal taxation. As the balanced budget condition (4) requires a transfer level equal to the expected tax revenue, there are no income or wealth effects that could increase the size of the required marginal compensation for risk taking. Thus it is clear that there is an increase in risk taking that produces an additional welfare gain beyond the gain from a reduction in uncertainty and inequality that would occur if people did not react to the imposition of the tax system.

Non-deductible Efforts

Consider now the other extreme case $\alpha = 0$. Here, the opportunity cost of effort occurs exclusively in the form of non-market income or leisure foregone, and non-market income or leisure is untaxed (labor income tax).

Inspecting (19) shows that the expected tax base now reduces to $\{m - E(L)\}$. Since m is a constant, the base is smaller the greater $E(L)$ and hence the larger the amount of risk taking as measured by σ_G; cf. equation (11). Thus the intuitive argument raised above suggests that the individual will want to deviate to the right from the social optimum Q' in Figure 5 in order to become a net recipient of public funds. There is an individual opportunity line cutting through the redistribution line at point Q' from below such that a higher indifference curve seems to be attainable by increasing σ and σ_G. Again, however, if everyone behaves that way, the public transfer p will have to be reduced, and the individual's position will be pushed downward, back to the redistribution line. The equilibrium V' where an indifference curve is tangent to the individual opportunity line, and where the point of the tangency is, in addition, located on the redistribution line, will be to the right of Q', possibly even to the right of the maximum as shown in the figure. This intuitive result is confirmed by

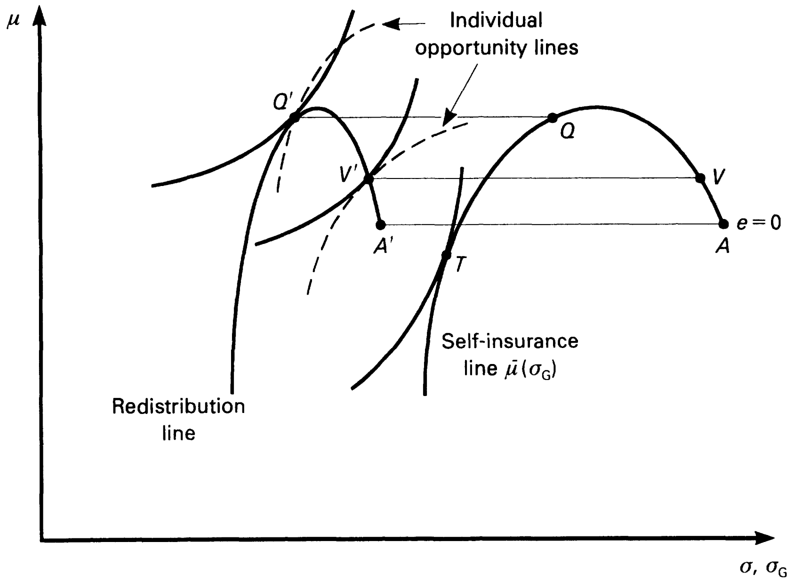

Fig. 5. Excessive inequality without deductibility of self-insurance (labor and income tax).

Proposition 4. *When self-insurance efforts are not tax deductible (as with a labor income tax), there will be some self-insurance effort but not enough: risk taking overshoots the social optimum, and too much inequality will result.*

Proof: In the case $\alpha = 0$, condition (23) becomes

$$\bar{\mu}'(\sigma_G) - i[\bar{\mu}(\sigma_G), (1-\tau)\sigma_G](1-\tau) = -\tau k. \tag{25}$$

Assume $0 < \tau < 1$ and let $r(\cdot)$ and $i(\cdot)$ denote the slopes of the self-insurance line and the indifference curve at the respective points (from Figure 5) named in the brackets. Let A be the end point of the self-insurance line where $e = 0$ and recall from (13) that $r(A) = -k$, k being a strictly positive parameter characterizing the distribution of θ (the state of the world). Recall furthermore from (17) that the social optimum is defined by $r(Q) - i(Q')(1-\tau) = 0$. Equation (25) defines a point V' on the redistribution line and its counterpart V horizontally to the right on the self-insurance line such that $r(V) - i(V')(1-\tau) = -\tau k$. Since $i \geq 0$, this implies $r(V) > r(A)$ which, because of the concavity of the self-insurance line, defines a point to the left of A. Moreover the concavity of the self-insurance line and the strict convexity of the indifference curves imply that $r(V) - i(V')(1-\tau) < 0$ can only hold true to the right of the social optimum. Thus $\sigma(Q') < \sigma(V') < \sigma(A')$ and $\sigma_G(Q) < \sigma_G(V) < \sigma_G(A)$, q.e.d.

The General Case

Since $\alpha=0$ implies too much and $\alpha=1$ too little inequality relative to the social optimum, there should be an intermediate value of α where the right amount of inequality will be generated. Equating the r.h.s. of (17) and of (23) gives

$$\bar{\alpha}(\sigma_G)=\frac{k}{\bar{\mu}'(\sigma_G)+k} \tag{26}$$

where $\bar{\alpha}(\sigma_G)$ is a function that indicates the level of α that equates the slope of the individual opportunity line with the slope of the redistribution line at a given level of σ_G. Let $\sigma_G(Q)$ be the socially optimal level of σ_G. Then $\alpha=\bar{\alpha}[\sigma_G(Q)]$ will ensure that the equilibrium coincides with the social optimum. Uniqueness of (26) and continuity of (23) imply that higher levels of α will induce too little, and lower levels too much, risk taking and inequality.

Note that the critical level of α depends on the size of the tax rate because the optimal amount of risk taking does so. From Proposition 2 and equation (18) it is known that $\sigma_G(Q)$ is a strictly increasing function of τ. Since $\bar{\mu}''\leq 0$, the optimal level of α increases with τ where $\bar{\mu}''<0$ and stays constant where $\bar{\mu}''=0$.

It is known from property (a) of the indifference system that $i(\mu,\sigma)=0$ when $\sigma=0$. In the limit, where $\tau\to 1$, this property and equation (17) imply that the socially optimal amount of risk taking, $\sigma_G(Q)$, converges to that value of σ_G where $\bar{\mu}$ has its maximum and $\bar{\mu}'=0$. The critical level of α will then converge towards unity such that, with any given $\alpha<1$, there will be too much risk taking.

In fact, when τ goes to unity, effort e approaches zero and $\sigma_G(V)$ approaches $\sigma_G(A)$, the maximum feasible value of σ_G. To see this, rewrite (23) in the form

$$i[\bar{\mu}(\sigma_G),\sigma_G(1-\tau)](1-\tau)=(1-\alpha)\left[\bar{\mu}'(\sigma_G)\frac{1-\alpha\tau}{1-\alpha}+\tau k\right]. \tag{27}$$

Clearly, $\tau\to 1$ implies that $\bar{\mu}'(\sigma_G)\to -k$, the condition characterizing point A. Conversely, if $\tau<1$, an equilibrium at point A is impossible. For one thing, the l.h.s. of (27) is now strictly positive since $i>0$ and $1-\tau>0$. For another, the r.h.s. of (27) would be negative if $\bar{\mu}'=-k$ and $\tau<1$. This becomes immediately obvious by differentiating the r.h.s. of equation (27) with regard to τ. As the derivative is positive (namely $+k$), $\tau<1$ implies a value less than zero.

These findings can be summarized as follows.

Proposition 5. *There is a critical value for the deductible proportion of self-insurance efforts greater than zero and smaller than one which generates an equilibrium with the optimal amount of risk taking and inequality. Higher values imply too little risk taking and inequality, lower values too much. The critical value is an increasing function of the tax rate and approaches unity as the tax rate does so.*

Proposition 6. *Assume that the deductible proportion of self-insurance effort is a constant less than one. Then there is always some self-insurance effort if the tax rate is less than one, but this effort will go to zero when the tax rate approaches one. In the limiting case $\tau \to 1$ there is no self-insurance effort and society will operate beyond the maximum of the self-insurance line where a higher average income could be reached by a reduction in pre-tax inequality.*

Proposition 6 confirms the scepticism of those who doubt that redistribution is an efficiency enhancing or even legitimate part of government activity. Since it is rarely the case in practice that all self-insurance efforts are tax deductible ($\alpha = 1$), it is unavoidable that an ongoing growth of the welfare state will eventually push the economy to the wrong side of its risk-return opportunity space and will tend to eliminate all self-insurance efforts. When the government absorbs all risks, excessive risk taking is the obvious consequence.

The disincentive effects of the welfare state may indeed be so strong that society on the whole loses from the existence of this state. Figure 6

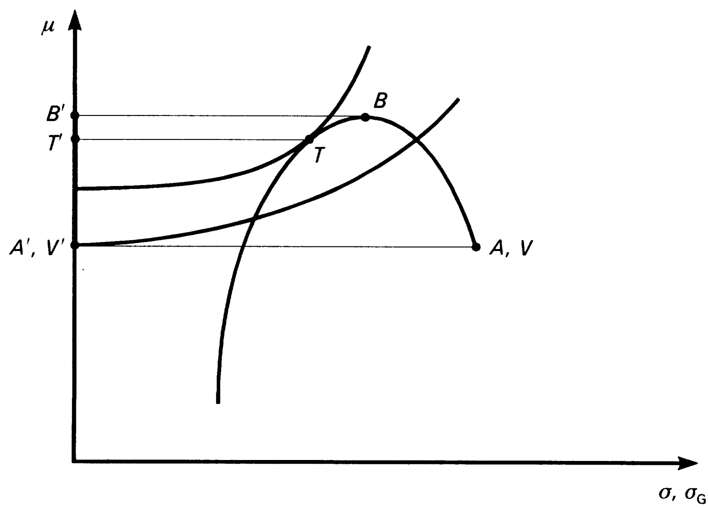

Fig. 6. The welfare loss from an overdrawn welfare state ($\tau \to 1$, $\alpha < 1$).

demonstrates such a possibility. Without any protection of the welfare state a point like T is chosen which is located to the left of the maximum of the self-insurance line. With full protection, the redistribution line converges to a straight line on the ordinate which extends from B' to A'. Since the perfect welfare state eliminates all incentives for self-protection, individuals choose the lowest point on the redistribution line ($V' = A'$). In the case at hand, this point is located on a lower indifference curve than the *laissez-faire* point T.

While α, the proportion of self-insurance effort consisting of the consumption of market resources, has been treated as exogenous thus far, the model does have implications for the case where the government can manipulate its size. To be on the safe side it would be better to choose a high value of α rather than a low one.[21] Truly detrimental effects can only occur when α is too small. When it is too high, the welfare gain from redistributional taxation will not be maximal, but at least there will be some gain. The insurance effect will in this case be fully present, and part of the potential welfare gain from risk taking can also be exploited. For practical tax systems this means that a move from capital income taxes towards cash flow taxes on capital is advisable, as are all measures which the optimal tax literature recommends for minimizing the labor-leisure distortion. In particular, the investment in human capital which may be the most important self-insurance activity in a market economy should be made fully tax deductible.

V. The Redistribution Paradox

How redistributive taxation will affect the equality of incomes is an old economic question. With any pre-tax income distribution, the variance of post-tax incomes is clearly reduced by redistributive taxation. However, people may react by taking more risks so that the pre-tax inequality rises. How strong is this countervailing effect? Is it possible that it offsets the primary effect?

Section IV showed, among other things, that the introduction of a linear redistribution system will increase the equilibrium pre-tax inequality. Before the impact of a tax rate change on the post-tax distribution can be considered, the marginal analogue of that result has to be proved.

[21] Alternatively it may be advisable to make only a fraction of the income losses tax deductible. However, as can be seen from equation (3), such policy does not offer an additional degree of freedom beyond what can be achieved with an appropriate choice of α and τ.

Proposition 7. *A marginal increase in the tax rate will increase the equilibrium inequality of pre-tax incomes.*

Proof: Implicit differentiation of (23) yields

$$\frac{\mathrm{d}\sigma_G}{\mathrm{d}\tau}=\frac{i_\sigma\sigma_G+[\beta/(1-\tau)]}{\gamma-\delta}>0 \tag{28}$$

where

$$\beta\equiv i-\alpha\bar{\mu}'(\sigma_G)+(1-\alpha)k, \tag{29}$$

$$\gamma\equiv i_\mu\bar{\mu}'(\sigma_G)+i_\sigma\cdot(1-\tau), \tag{30}$$

$$\delta\equiv\bar{\mu}''(\sigma_G)(1-\alpha\tau)/(1-\tau). \tag{31}$$

Here, the indifference curve slope i and its derivatives i_μ and i_σ are functions of μ and σ, where $\mu=\bar{\mu}(\sigma_G)$ and $\sigma=(1-\tau)(\sigma_G)$.

To sign (28) consider first the numerator. It is clearly positive. For one thing, property (d) of the indifference curves ensures that $i_\sigma\sigma_G>0$. For another, if $\alpha\bar{\mu}'-(1-\alpha)k$ is subtracted from both sides of equation (23), it follows after a few algebraic manipulations that

$$\beta=(\bar{\mu}'+k)(1-\alpha)/(1-\tau). \tag{32}$$

Since it is known from Proposition 6 and the preceding discussion that $\bar{\mu}'+k$ is positive and will only in the limiting case $\tau\to 1$ approach zero, it follows that

$$\beta\begin{Bmatrix}>\\=\end{Bmatrix}0 \qquad \text{for } \alpha\begin{Bmatrix}<\\=\end{Bmatrix}1 \text{ and } \tau<1, \tag{33}$$

a result that will also be needed below.

Consider the denominator next. The terms γ and δ measure the marginal changes of the slope of the indifference curve and the individual opportunity line, respectively, brought about by a rightward movement *along the redistribution line* (and along neither a given indifference curve nor a given individual opportunity line). It can be shown that $\gamma-\delta>0$ is a stability condition for the equilibrium and that the existence of a stable equilibrium is ensured.[22] The correspondence principle therefore implies that $\mathrm{d}\sigma_G/\mathrm{d}\tau>0$. **Q.e.d.**

Consider now post-tax incomes. Since $\sigma=(1-\tau)\sigma_G$ (from (9) and (21)) is the standard deviation of the income distribution net of taxes and public

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[22] The complete proof is contained in the Appendix of an earlier version of this paper; see NBER WP 4856, 1994 and CES WP 65, 1994.

transfers, it holds that

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}=(1-\tau)\frac{\mathrm{d}\sigma_{\mathrm{G}}}{\mathrm{d}\tau}-\sigma_{\mathrm{G}}. \tag{34}$$

Using (28), (29) and (30) this expression can be transformed to

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}=\frac{\beta-\sigma_{\mathrm{G}}i_{\mu}\bar{\mu}'(\sigma_{\mathrm{G}})+\bar{\mu}''(\sigma_{\mathrm{G}})[(1-\alpha\tau)/(1-\tau)]}{\gamma-\delta}. \tag{35}$$

The sign of (35) is ambiguous. Since $\gamma-\delta>0$, it equals the sign of the numerator.

Note first that $\mathrm{d}\sigma/\mathrm{d}\tau<0$ if $\bar{\mu}''$ is sufficiently strongly negative. A negative sign for $\bar{\mu}''$ indicates a curved self-insurance line and decreasing returns to risk taking. With a strongly negative value of $\bar{\mu}''$, the scope for individual reactions to a tax increase is small, and obviously the direct effect of a tax increase dominates.

A more interesting possibility is the one where $\bar{\mu}'$ is a positive constant in the relevant range such that $\bar{\mu}''=0$. In this case, equation (35) simplifies to

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}=\frac{\beta-\sigma_{\mathrm{G}}i_{\mu}\bar{\mu}'}{\gamma-\delta} \qquad \text{for } \bar{\mu}'=\text{const.} \tag{36}$$

Recalling property (e) of the indifference curve system and (33) this expression can easily be interpreted.

Proposition 8. *Suppose there are constant returns to risk taking in the relevant range. Then, with decreasing absolute risk aversion ($i_{\mu}<0$), an expansion of the redistribution system will imply an equilibrium with more post-tax inequality. The same will be true with constant absolute risk aversion ($i_{\mu}=0$) provided that less than 100 per cent of self-insurance efforts are tax deductible. With constant absolute risk aversion and full deductibility of self-insurance efforts, the equilibrium post-tax inequality will not be affected by the tax rate.*[23]

[23] The proposition is related to a result that had been derived in another context by Atkinson and Stiglitz (1980, p. 119). These authors studied redistributive taxation in the context of the standard two-asset portfolio problem (where the (μ,σ) trade-off is automatically constant) and found that taxation increases "private risk taking" if the wealth elasticity of demand for the risky asset is positive. There is also a similarity with a problem in traffic regulation where artificial impediments to traffic (like road bumps) lead to an overreaction of drivers, implying an increase in safety despite the deterioration of driving conditions; see Risa (1994). I am grateful to Kjell Erik Lommerud for leading me to this paper.

Proposition 8 describes a redistribution paradox because it specifies conditions under which the primary effect of increased taxes on equality will be overcompensated by the secondary effect of increased risk taking. This gives a deeper meaning to the statement made in the introduction that the risk taking effect of redistributive taxation may be more important than the insurance effect. In the cases considered, people transform more than 100 per cent of the increase in equality through redistributive taxation into income increases. Redistributive taxation does not improve the distribution of the pie's slices, but it makes the pie bigger.

An intuitive explanation of Proposition 8 can be given using Figure 7. This figure incorporates the cases of constant and decreasing absolute risk aversion and assumes that α equals unity (full deductibility of effort). The self-insurance line is linear in the relevant range, and so is the redistribution line. The equilibrium is characterized by a point on the redistribution line which is also a point of tangency between an indifference curve and the individual opportunity line. Depending on the level of government transfers, the latter can have a continuum of alternative positions. For the case at hand ($\alpha = 1$), it is known from (24) that the individual opportunity line has the same slope as the self-insurance line. The possible positions of the

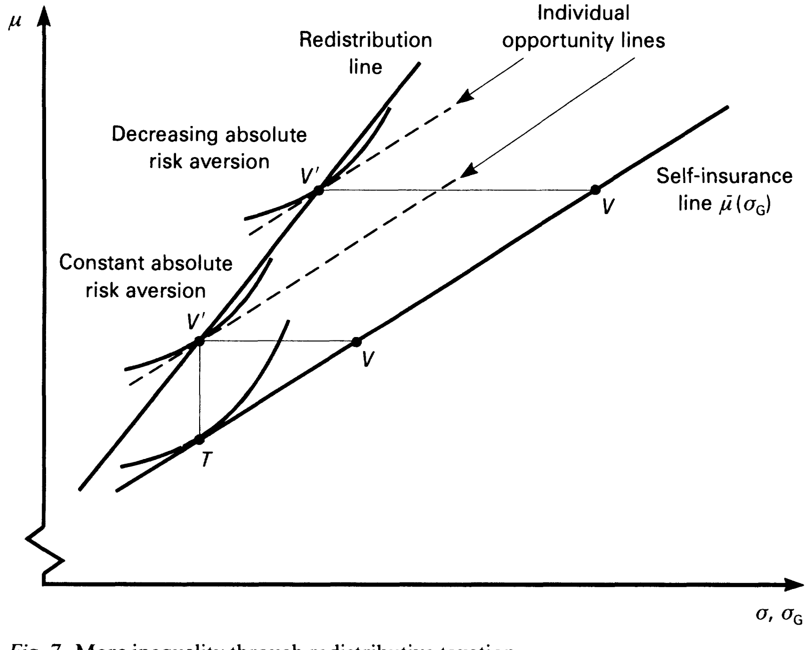

Fig. 7. More inequality through redistributive taxation.

individual opportunity line can therefore be constructed by parallel shifts of the self-insurance line to the left. When absolute risk aversion is constant, the indifference curve slope stays constant when μ increases, given σ ($i_\mu = 0$). The equilibrium point V' on the redistribution line will therefore be vertically above the *laissez-faire* point T, while the point characterizing the pre-tax distribution shifts from T to V on the self-insurance line. The advantage of the protection that the redistribution scheme offers is entirely translated into a higher average income.

On the basis of this neutrality result, it is easy to see under which conditions the equilibrium point V' will be to the right of the *laissez-faire* point T. A first and obvious possibility is the case where, vertically above T, the indifference-curve slope is lower than at T. This case prevails under decreasing absolute risk aversion. For any given level of post-tax inequality, pre-tax inequality and average income rise with an introduction of the redistribution scheme. The rise in average income lowers the required marginal compensation of risk taking, $i(\mu, \sigma)$. The actual marginal compensation perceived by the individual, $\bar{\mu}'$, is constant, on the other hand. Hence, an equilibrium with a higher level of post-tax inequality will result. Figure 5 illustrates this with the upper of the two solution points labelled V'.

The second reason (not shown in the figure) for an equilibrium with a higher inequality in post-tax incomes is incomplete deductibility of self-insurance efforts ($\alpha < 1$). Incomplete deductibility means that the decision maker perceives an additional incentive to reduce his effort and to move along the self-insurance line towards higher values of pre-tax inequality. In Figure 7, the individual opportunity line would have a higher slope than the self-insurance line and so the solution point V' would be to the right of T even in the case where absolute risk aversion is constant ($i_\mu = 0$).[24]

The conditions under which the redistribution paradox emerges are not implausible. From an empirical point of view, there can be little doubt that decreasing absolute risk aversion and less than full deductibility of self-insurance efforts are realistic assumptions. So the assumption of constant returns to risk taking is crucial. With the specifications of this model, this assumption is only a limiting case. However, other model specifications may rather give the impression that constant returns to scale are an intermediate case in the spectrum of possibilities. For example, when there are decreasing returns to self-insurance while, at the same time, it is possible for an agent to add up independent income risks, then it is entirely unclear whether there will be increasing or decreasing returns to risk taking, since adding up independent income risks in itself implies increas-

[24] This effect is operative even when $\bar{\mu}' = 0$. Cf. the next section, in particular equation (40).

ing returns to risk taking. Increasing returns to risk taking would strengthen the mechanism underlying the redistribution paradox.

VI. The Optimal Welfare State

Up till now it has been assumed that the government is a fairly passive agent satisfying itself with adjusting the public transfer so as to balance the government budget. What if the government chooses the tax rate so as to maximize the representative individual's expected utility? What are the characteristics of the optimal welfare state?

To make the problem interesting it has to be assumed that $\alpha < 1$ so that at least some moral hazard effect is present. With $\alpha = 1$ the model would predict an optimal tax rate of one, since successive tax increases would always generate welfare increasing insurance and risk taking effects. Assuming that at least part of the agent's effort results in a loss of non-market income (i.e., leisure or goods produced and consumed during "leisure" time) is common to the optimal tax literature. Without this assumption the optimal tax problem would not yield an intermediate solution.

The problem of optimal taxation is illustrated in Figure 8. For every tax rate τ, there is an equilibrium as described by equation (23). Starting from the *laissez-faire* point T, an increase in the tax rate will therefore induce a movement to the right along the self-insurance line (Proposition 7). In addition, the tax increase will move the redistribution line (cf. Figure 1) to the left. The net effect on the equilibrium combinations of μ and σ attainable through successive tax rate changes is illustrated by the arrowed curve in Figure 8 which will be called the "equilibrium line". It is known from Proposition 6 that the equilibrium line ends at point A' on the ordinate when the tax rate approaches one. (A' is the counterpart of A on the self-insurance line which is characterized by an absence of self-insurance effort.) The optimal tax rate is determined by a point like Z' where an indifference curve is tangent to the equilibrium line. Z' and its counterpart Z on the self-insurance line coincide with points like V' and V in Figure 5 if that figure is drawn for the optimal tax rate. The magnitude of the tax rate equals the distance $Z'Z$ relative to the distance between Z and the ordinate.

Let $\bar{\sigma}_G(\tau)$ be a function that summarizes the relationship between the equilibrium amount of pre-tax inequality and the tax rate as calculated with (28). Then the problem of optimal taxation can be stated as follows:[25]

[25] This formulation incorporates the government budget constraint through the assumption $\mu = \bar{\mu}(\sigma_G)$.

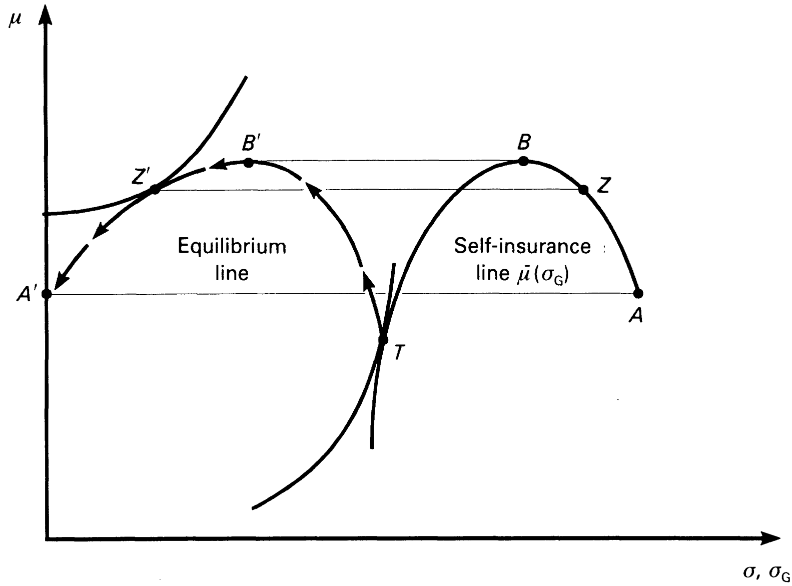

Fig. 8. One version of the optimal tax problem.

$$\max_{\tau} U(\mu, \sigma) \quad \text{s.t.}\ \mu = \bar{\mu}(\sigma_G), \quad \sigma = (1-\tau)\sigma_G, \quad \sigma_G = \bar{\sigma}_G(\tau). \tag{37}$$

Let $(\mathrm{d}U/\mathrm{d}\tau)/U_\mu$ denote the tax-induced welfare change in terms of certainty equivalents or what Atkinson (1970) called "equally distributed equivalent incomes". Differentiation of $U(\mu, \sigma)$ yields:

$$\frac{\mathrm{d}U/d\tau}{U_\mu} = i\sigma_G + \bar{\sigma}'_G(\tau)[\bar{\mu}'(\sigma_G) - i(1-\tau)] \tag{38}$$

where $i = i(\mu, \sigma)$ is the indifference curve slope as defined in (15). A change in the tax rate generally alters μ and σ. The r.h.s. of equation (38) evaluates these alterations. The term $i\sigma_G$ is the direct gain from redistribu- $\bar{\sigma}'_G(\tau)[\bar{\mu}' - i(1-\tau)]$ is the welfare change resulting from the increase in risk taking: it consists of a change in per capita income, $\bar{\sigma}'_G\bar{\mu}'$, and a change in post-tax inequality evaluated at the individual's "price of risk" (the indifference curve slope), $\bar{\sigma}'_G i(1-\tau)$.

From (17) it is known that, if risk taking is at the socially optimal level given the tax rate, then $\bar{\mu}' - i(1-\tau) = 0$. As this includes the *laissez-faire* situation where $\tau = 0$, the first bit of redistributive taxation must increase welfare through the direct gain from redistribution; i.e., $(\mathrm{d}U/\mathrm{d}\tau)/U_\mu = i \cdot \sigma_G > 0$ at $\tau = 0$. At $\tau = 1$, according to Proposition 6, effort is zero so that $\bar{\mu}' = -k < 0$. Since, in addition, $i = i(\mu, \sigma) = 0$, from property (a) of the indifference curve system, the marginal increase in welfare approaches $(\mathrm{d}U/\mathrm{d}\tau)/U_\mu = -\bar{\sigma}'_G(\tau)k < 0$ as $\tau \to 1$. This implies that there is an interior solution for the optimal tax rate such as the one illustrated in Figure 8.

In the optimum, it is necessary that $(\mathrm{d}\mu/\mathrm{d}\tau)/U_\mu = 0$, which means that the welfare gain from the insurance effect is outweighed by a welfare loss resulting from excessive risk taking:

$$i\sigma_G = -\bar{\sigma}'_G(\tau)[\bar{\mu}'(\sigma_G) - i(1-\tau)] > 0. \tag{39}$$

Since $i\sigma_G > 0$ and $\bar{\sigma}'_G > 0$, it is necessary for (39) to be true that $\bar{\mu}'/(1-\tau) < i$. A comparison with (17) shows that this condition implies an equilibrium point on the redistribution line to the right of the constrained social optimum Q'. The result can be summarized as follows.

Proposition 9. *When self-insurance efforts are not fully tax deductible, there is an interior solution for the socially optimal tax rate. In the optimum, risk taking and inequality overshoot the constrained social optimum, given a tax rate at the level of the optimal rate.*

The overshooting of risk taking may be substantial. In the case considered in Figure 8, it even implies moving to a point to the right of the maximum of the self-insurance line, where the marginal return to risk taking is negative.

Figure 8 does not, however, depict the only possible case. An alternative possibility is illustrated in Figure 9. Here the equilibrium line performs a loop, and the optimal size of the redistributive system is found before the maxima of the self-insurance line and the equilibrium line are reached. The solution is now located in the range of positive marginal returns to risk taking (albeit still in the range where the marginal return to risk taking is unable to compensate for the resulting marginal increase in inequality).

Since σ_G is a monotonically increasing function of τ, and μ is a concave function of σ_G, a necessary and sufficient condition for a loop in the equilibrium line is that, at the maxima of the two curves, a redistribution paradox is present; i.e., it is necessary that, in the neighborhood of the point where $\bar{\mu}' = 0$, post-tax inequality rises with an increase in the tax rate.

To check whether and under what conditions this can be the case, insert (29), (30) and (31) into (35). If $\bar{\mu}' = 0$, this expression becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{i + (1-\alpha)k + \bar{\mu}''[(1-\alpha\tau)/(1-\tau)]}{i_\sigma(1-\tau) - \bar{\mu}''[(1-\alpha\tau)/(1-\tau)]}. \tag{40}$$

Equation (40) shows that the curvature of the self-insurance line, $|\bar{\mu}''|$, is essential for the existence of a loop. If the self-insurance line is sufficiently

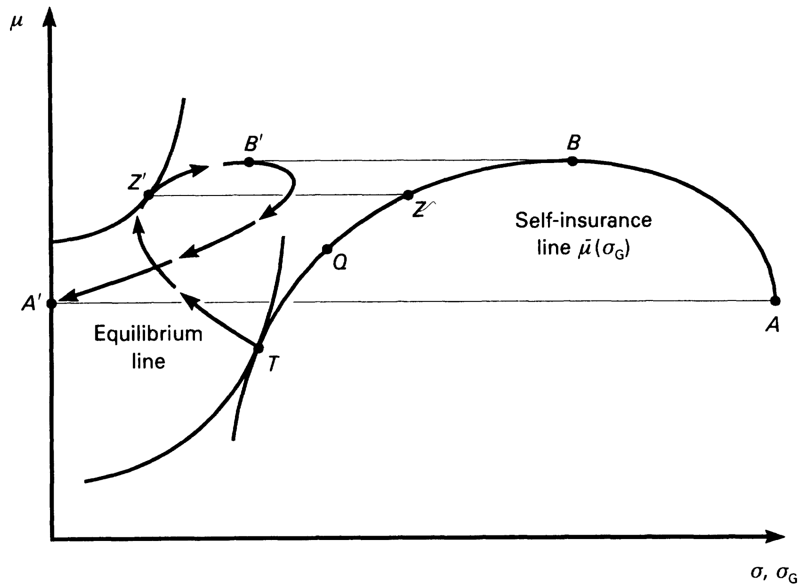

Fig. 9. Optimal taxation and the redistribution paradox.

curved, then $d\sigma/d\tau < 0$ and there will be no loop. If it is sufficiently flat, there will be one. General continuity arguments imply that $d\sigma/d\tau$ will be strictly positive in the neighborhood of the maximum of $\bar{\mu}(\sigma_G)$ if $|\bar{\mu}''|$ stays sufficiently small in that neighborhood.

The interesting aspect of the solution illustrated in Figure 9 is that the redistribution paradox is present when the size of the welfare state has been optimized. A marginal increase in the tax rate increases average income, but this advantage is outweighed by an increase in post-tax inequality.

The nature of the two kinds of solution becomes apparent when equation (34) is inserted into (39). The resulting version of the optimality condition,

$$\frac{d\sigma_G}{d\tau}\,\bar{\mu}'(\sigma_G) = i\,\frac{d\sigma}{d\tau}, \tag{41}$$

shows that $\bar{\mu}'$ and $d\sigma/d\tau$ will have the same sign. In the case depicted in Figure 9, the common sign is positive; in the case depicted in Figure 9 it is negative. The following proposition emphasizes the interesting aspects of this result.

Proposition 10. *With an optimal size of the redistributive tax system, one of the two following conditions will hold. Either the economy operates at a point on its self-insurance line where, given the tax rate, more inequality results in a smaller average income, or more redistribution causes more inequality in post-tax incomes and a higher average income.*

Although it contradicts popular views, Proposition 10 is a very natural and straightforward implication of a preference for equality when — as in the present model — the inequality of pre-tax incomes is an increasing function of the tax rate. Obviously, in the optimum, a marginal tax change must not induce adverse movements of average income and post-tax inequality for, if it did, a tax reform could be designed that increases welfare. Instead a marginal tax change must either decrease post-tax inequality and average income or have precisely the reverse effect. In the former case, a fall in average income coincides with an increase in pre-tax inequality; thus, given the redistribution scheme, the economy's technology implies a positive relationship between the size of the pie and the equality in the distribution of its slices. In the latter case, more redistribution increases the pie, but makes its distribution more unequal.

VII. Concluding Remarks

This analysis has countered popular views concerning the role of the welfare state. It is not true that the welfare state will always reduce inequality and it is not true that it will always make the pie smaller. The paper has studied cases confirming the conventional wisdom, but it has also emphasized the important role of the welfare state as a device for stimulating risk taking, thereby liberating productive forces and increasing aggregate income. Under constant returns to risk taking, the stimulus is likely to be so strong that more than 100 per cent of the risk consolidating effect of the welfare state is being translated into an income increase. Thus, the welfare state would make people richer, but not necessarily more equal and not necessarily happier. In fact, the moral hazard effect resulting from a likely imperfect deductibility of individual effort produces a welfare loss which needs to be subtracted from the welfare gain which under ideal circumstances could be achieved.

One of the less satisfactory aspects of this paper concerns the way the risk-return trade-off has been modeled. There are certainly alternatives to the self-insurance specification chosen here. Pigou (1932) once called risk a "forgotten factor of production", alluding to the prominent role classical economists had attributed to risk taking. Indeed it seems that theoretical and empirical research on the productive effects of risk taking would be highly rewarding.

References

Ahsan, S. M.: Progression and risk-taking. *Oxford Economic Papers 26*, 318–28, 1974.

Ahsan, S. M.: Taxation in a two-period temporal model of consumption and portfolio allocation. *Journal of Public Economics 5*, 337–52, 1976.

Allingham, M. G.: Risk taking and taxation. *Zeitschrift für Nationalökonomie 32*, 203–24, 1972.

Atkinson, A. B.: On the measurement of inequality. *Journal of Economic Theory 2*, 244–63, 1970.

Atkinson, A. B. & Stiglitz, J.: *Lectures on Public Economics.* McGraw-Hill, New York, 1980.

Bamberg, G. & Richter, W. F.: The effects of progressive taxation on risk-taking. *Zeitschrift für Nationalökonomie 44*, 93–102, 1984.

Buchanan, J. M. & Tullock, G.: *The Calculus of Consent.* University of Michigan Press, Ann Arbor, 1962.

Christiansen, V.: Subsidization of risky investment under income taxation and moral hazard. Warwick Economic Research Paper 357, 1990.

Diamond, P. H., Helms, L. J. & Mirrlees, J. A.: Optimal taxation in a stochastic economy. *Journal of Public Economics 14*, 1–29, 1980.

Diamond, P. H. & Mirrlees, J. A.: A model of social insurance with variable retirement. *Journal of Public Economics 10*, 295–336, 1978.

Domar, E. & Musgrave, R. A.: Proportional income taxation and risk-taking. *Quarterly Journal of Economics 58*, 388–422, 1944.

Eaton, J. & Rosen, H. S.: Optimal redistributive taxation and uncertainty. *Quarterly Journal of Economics 95*, 1980, 357–64, 1980.

Ehrlich, I. & Becker, G. S.: Market insurance, self-insurance, and self-protection. *Journal of Political Economy 80*, 623–48, 1972.

Feldstein, M.: The effects of taxation on risk taking. *Journal of Political Economy 77*, 755–64, 1969.

Friedman, M.: Choice, chance, and the personal distribution of income. *Journal of Political Economy 61*, 277–90, 1953.

Gordon, R. H.: Taxation of corporate capital income: Tax revenues versus tax distortions. *Quarterly Journal of Economics 100*, 1–27, 1985.

Gordon, R. H. & Varian, H.: Intergenerational risk sharing. *Journal of Public Economics 37*, 1988, 185–202, 1988.

Harsanyi, J. C.: Cardinal utility in welfare economics and the theory of risk-taking. *Journal of Political Economy 61*, 434–5, 1953.

Harsanyi, J. C.: Cardinal welfare, individualistic ethics and interpersonal comparisons of utility. *Journal of Political Economy 63*, 309–21, 1955.

Kanbur, R.: Of risk taking and the personal distribution of income. *Journal of Political Economy 87*, 769–97, 1979.

Kaplow, L.: A note on taxation as social insurance for uncertain labor income. NBER WP 3708, 1991.

Kaplow, L.: Income tax deductions for losses as insurance. *American Economic Review 82*, 1013–17, 1992.

Konrad, K.: Risk taking and taxation in complete capital markets. *Geneva Papers on Risk and Insurance Theory 16*, 167–77, 1991.

Konrad, K.: *Risikoproduktivität.* Springer, Berlin, Heidelberg, New York, 1992.

Meyer, J.: Two-moment decision models and expected utility maximization. *American Economic Review 77*, 421–30, 1987.

Pigou, A. C.: *The Economics of Welfare.* McMillan, London, 1932.

Rawls, J.: *A Theory of Justice.* Harvard University Press, Cambridge, MA, 1971.

Risa, A. E.: Adverse incentives from improved technology: Traffic safety and regulation in Norway. *Southern Economic Journal 60,* 844–57, 1994.

Rochet, J.-C.: Incentives, redistribution and social insurance. *Geneva Papers on Risk and Insurance Theory 16,* 143–65, 1991.

Sandmo, A.: Portfolio choice, asset demand and taxation. *Review of Economic Studies 44,* 369–79, 1977.

Sinn, H.-W.: Die Grenzen des Versicherungsstaates. Theoretische Bemerkungen zum Thema Einkommensumverteilung, Versicherung und Wohlfart. In H. Göppl & R. Henn (eds.), *Geld, Banken und Versicherungen,* Athenäum, Königstein; 907–28, 1981; reprinted in G. Rolf, P. B. Spahn & G. Wagner (eds.), *Sozialvertrag und Sicherung — Zur ökonomischen Theorie staatlicher Versicherungs- und Umverteilungssysteme,* Campus, Frankfurt and New York, 65–84, 1988.

Sinn, H.-W.: *Economic Decisions under Uncertainty.* North Holland, Amsterdam, New York and Oxford, 1983.

Sinn, H.-W.: Risiko als Produktionsfaktor. *Jahrbücher für Nationalökonomie und Statistik 201,* 557–71, 1986.

Sinn, H.-W.: Two-moment decision models and expected utility maximization: Comment. *American Economic Review 79,* 601–2, 1989.

Sinn, H.-W., Expected utility, μ, σ preferences, and linear distribution classes: A further result. *Journal of Risk and Uncertainty 3,* 277–81, 1990.

Stiglitz, J. E.: The effects of income, wealth, and capital gains taxation on risk taking. *Quarterly Journal of Economics 83,* 263–83, 1969.

Varian, H. R.: Redistributive taxation as social insurance. *Journal of Public Economics 14,* 49–68, 1980.