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Dynamic Equilibria

by John McMillan and Hans-Werner Sinn

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Oligopolistic extraction of a common-property resource: Dynamic equilibria

JOHN McMILLAN and HANS-WERNER SINN*

1. Introduction

The speed with which firms choose to extract a natural resource depends crucially on the value the firms attach to the unextracted resource. Under well-defined property rights, abstracting from imperfections in the final market for the firms' outputs, the firms will extract at the socially optimal rate. When the resource is owned in common and entry into the industry is free, the firms have no incentive to conserve the resource because they know that newcomers to the industry will extract immediately any unit of the resource that can be extracted with immediate profits. This case has been thoroughly analysed in the literature.[1] When the resource is owned in common but the number of firms is fixed (perhaps because each extracting firm must have a lease to the property from which the resource is extracted), the firms have some incentive to conserve the resource: they know that immediate profitability does not necessarily result in immediate extraction by rivals. It does not follow, however, that this incentive to conserve the resource is strong enough to generate a socially optimal extraction rate. Each unit of the resource which a firm chooses not to extract today may in part be extracted by a rival film tomorrow; thus, even without free entry, the firms' valuation of the unextracted

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[1]See, for example, Berck (1979). Dasgupta and Heal (1979, ch. 3), Gordon (1954), Hoel (1978), and Weitzman (1974).

Essays in the Economics of Exhaustible Resources, edited by M. C. Kemp and N. V. Long

resource may be too low and the firms may extract the resource too quickly. (The belief that common-property resources are extracted too quickly has motivated much of the regulation of the petroleum industry: see McDonald (1971, chs. 1–3) and Watkins (1977).)

This essay investigates dynamic equilibria for an oligopolistic industry with a given number of firms exploiting a common-property non-renewable resource. It excludes the problem of market imperfections through the assumption of a constant-elasticity demand curve[2] and thus concentrates on the distortions due to the common-pool aspect.

Several recent studies have examined the dynamics of the exploitation of non-renewable common-property resources by an oligopolistic industry. Bolle (1980) considered the case of a common stock of a resource to which several countries have access. Dasgupta and Heal (1979, ch. 12), Kemp and Long (1980), Khalatbari (1977), and Sinn (1983) analysed the problem of oil-well owners who have the right to extract the oil located under their own properties: the oil is in a single pool underground, and seeps from one holding to another at a speed dependent on the relative sizes of the stocks currently under each property.

In modelling dynamic oligopoly, some choice of equilibrium concept must be made. A natural candidate is a dynamic analogue of the static equilibrium concept introduced by Cournot: each firm makes its decisions under the assumption that its rivals' actions are not affected by its own actions. Unfortunately, for dynamic common-property problems the meaning of Cournot-type behaviour is ambiguous. One possible Cournot-type assumption (adopted by Bolle (1980) and Kemp and Long (1980)) is that each agent believes its rivals will follow a particular time path of rates of extraction, regardless of its own actions. An alternative Cournot-type assumption (used by Sinn (1983)) is that each firm believes that, regardless of its own actions, its rivals will extract in such a way as to generate a particular time path of the stock of the resource. A third possibility (Khalatbari (1977), Dasgupta and Heal (1979)) is that each firm believes that its rivals both maintain a given time path of sales and maintain a given time path of the stock of the resource. The qualitative predictions of the models are sensitive to the choice of equilibrium concept: the models of Dasgupta and Heal, Khalatbari, and Sinn predict over-exploitation of the resource, while the Bolle and Kemp–Long models predict Pareto-optimal extraction rates. Thus, the decision as to

[2]Isoelastic demand ensures that there is no distortion due to oligopoly power when the commodity is sold on the market; see Stiglitz (1976) and Weinstein and Zeckhauser (1975). On the oligopolistic distortions with non-constant elasticity in a model with no common-property aspect, see Lewis and Schmalensee (1980).

whether or not there is a role for government intervention in common-property markets is dependent upon which equilibrium concept is thought to be appropriate.

In section 2 we examine a more general concept of equilibrium for the dynamic common-property problem by allowing firms to have arbitrary conjectures about their rivals' reactions. Then many equilibria are possible, including in particular the three Cournot-type equilibria.

Fellner (1949) criticized Cournot's equilibrium concept because it required firms' actions to be "right for the wrong reasons": at equilibrium, the firms act consistently but under incorrect assumptions about their rivals' reactions. In a formally static model such as Cournot's, this concept of equilibrium is not unreasonable; if the game is only played once, the incorrectness of conjectures may not be revealed. In an explicitly dynamic model, Fellner's criticism has more force. In a dynamic context, it seems likely that, if conjectures are incorrect, this incorrectness will be revealed, either during the initial adjustments on the approach to equilibrium, or by occasional accidental or experimental deviations after equilibrium has been reached. As an alternative to a dynamic Cournot-type equilibrium, in section 3 we define a rational-expectations equilibrium to be an equilibrium in which firms' conjectures are locally correct.

The model is developed for the case of a common pool of a resource, to the whole of which each firm in the industry has access. The results will be compared with results already reported in the literature. Since many of these existing results refer to the different but related problem of oil in a reservoir seeping from one individually owned property to another, it is necessary to show that the two problems are indeed comparable; this is done in section 4. Section 5 offers concluding comments.

2. Conjectural equilibria

An industry consisting of n firms ($n \geq 2$) exploits a common pool of a non-renewable resource. There is no entry into or exit from the industry. Each firm knows the industry's instantaneous demand function $P(R)$, $P' < 0$, where $R(t) = \sum_{i=1}^{n} R_i(t)$ is the total amount extracted and sold at time t and $R_i(t)$ is the amount extracted and sold by firm i. Assume, moreover, that the demand function is isoelastic, so that $\eta \equiv -P/(RP') > 0$ is constant. Each firm knows the size of the stock of the resource, $S(t)$.

As is standard in such models, extraction is assumed to be costless.[3] Firm i chooses an extraction plan seeking to maximize the discounted value of its stream of future profits. Since the market price, and therefore firm i's profit, at time t depends upon all the other firms' extraction rates, without some prediction of its rivals' actions firm i's optimization problem is not well defined. Denote firm i's conjecture about the total extraction rate of the other firms by $R^c_{-i}(t)$. Assume firm i's conjectures are of the form:

$$R^c_{-i}(t) = \alpha(t) + \beta S(t), \tag{1}$$

where β is constant, $0 \leq \beta < \infty$, and $\alpha(t) + \beta S(t) \geq 0$. The term $\alpha(t)$ in firm i's conjectures indicates that firm i believes that, in part, its rivals' extraction rate is autonomous. The term $\beta S(t)$ reflects firm i's belief that a change in the size of the resource stock will cause a change in the rivals' extraction rate; β will be called the "conjectural parameter".

Thus, firm i believes that the resource stock will change at the rate

$$\begin{aligned} \dot{S}^c(t) &= -R^c(t) \\ &= -(R_i(t) + R^c_{-i}(t)) \\ &= -(R_i(t) + \alpha(t) + \beta S(t)). \end{aligned} \tag{2}$$

Given $S(0) = S_0 > 0$, firm i's objective is to choose, subject to (2), an extraction plan $R_i(t)$ with $R_i(t) \geq 0$, to maximize

$$\int_0^\infty P(R^c(t))R_i(t)e^{-rt}dt, \tag{3}$$

where $r > 0$ is the market rate of interest.

The Hamiltonian is:

$$\mathcal{H}_i = \exp(-rt)[P(R^c)R_i - \lambda_i R^c]. \tag{4}$$

Under the assumption of an interior solution, necessary conditions are, from $\partial\mathcal{H}_i/\partial R_i = 0$;

$$\lambda_i = P(R^c)\left(1 - \frac{R_i}{\eta R^c}\right), \tag{5}$$

where

$$\eta > R_i/R^c. \tag{6}$$

[3]The assumption of costless extraction is not essential. The analysis remains valid if there is a constant average cost of extraction k and if η is redefined as $\eta \equiv -(P-k)/P'R$, where η is constant.

Also, from $\partial(\exp(-rt)\lambda_i)/\partial t = -\partial\mathcal{H}_i/\partial S$, and from (1):

$$\dot{\lambda}_i - r\lambda_i = -\beta[P'(R^c)R_i - \lambda_i]. \tag{7}$$

Let $\hat{x}$ denote $\dot{x}/x$. Then, by the use of (5), (7) becomes:

$$\hat{\lambda}_i = r + \beta\left[1 + \frac{1}{\eta R^c/R_i - 1}\right] \tag{8}$$

The transversality condition is:

$$\lim_{t\to\infty} \exp(-rt)\lambda_i(t)S(t) = 0, \tag{9}$$

which requires, because of $\hat{\lambda}_i \geq r$ (from $\beta \geq 0$, (6), and (8)):[4]

$$\lim_{t\to\infty} S(t) = 0. \tag{10}$$

Conditions (2), (5), (7), and (10) determine firm i's extraction path, $R_i(t)$.

Each firm is assumed to make its decision about $R_i(t)$ in this way. In a conjectural equilibrium, firm i's conjecture about the total of its rivals' extraction paths, $R^c_{-i}(t)$, must be equal to the rivals' actual total extraction path found as the solution to such maximization problems: $R_{-i}(t) = \sum_{j \neq i} R_j(t)$. Thus, in equilibrium $R^c_{-i}(t) = R_{-i}(t)$ for all $t \in (0, \infty)$.

We consider only symmetric equilibria, so that in equilibrium $R/R_i = n$. Condition (6) therefore becomes:

$$\eta n > 1. \tag{11}$$

Clearly, $R/R_i = n$ implies that $(1 - 1/(\eta R/R_i))$ is constant; hence in equilibrium, (5) and (8) imply:

$$-\hat{R}/\eta = \hat{\lambda}_i, \tag{12}$$

so that:

$$\hat{R} = -\rho, \tag{13}$$

where

$$\rho \equiv \eta\left[r + \beta\left(1 + \frac{1}{\eta n - 1}\right)\right]. \tag{14}$$

[4]This is the point at which it is necessary to assume $\beta \geq 0$ (that is, each firm believes that part of every unit of the resource it leaves unextracted will be extracted by its rivals). If β could be negative, (10) would not follow from (9).

Since (2) and (10) imply that $S(t)=\int_t^\infty R(\tau)\,\mathrm{d}\tau$ and since, because of (13), $\int_t^\infty R(\tau)\,\mathrm{d}\tau=R(t)/\rho$ we find that:

$$\hat{S}=-R/S=-\rho. \tag{15}$$

This equation also implies that $\rho/n=R_i/S$ for all i, i.e. that ρ/n is the single firm's actual rate of extraction per unit of stock.

In an equilibrium, the transversality condition (9) becomes:

$$\lim_{t\to\infty}\exp(-rt)\lambda_i(0)\exp(t\rho/\eta)S(0)\exp(-\rho t)=0, \tag{16}$$

when use is made of (12), (13), and (15). Inserting ρ from (14), and after some manipulations, we can write this as:

$$\eta r+(\eta-1)\beta\left(1+\frac{1}{n\eta-1}\right)>0. \tag{17}$$

Given, from (6), $\eta n>1$, and given $\beta\geq 0$, (17) is satisfied if and only if either

$$\eta\geq 1 \tag{18a}$$

or

$$\frac{1}{n}<\eta<1 \quad \text{and} \quad \beta<\frac{\eta r}{(1-\eta)\left(1+\dfrac{1}{n\eta-1}\right)}. \tag{18b}$$

The crucial result of this model is contained in eq. (14); the right-hand side of this will be called the extraction function. Rewrite (14) as:

$$\rho=a+b\beta, \tag{19}$$

where

$$a\equiv\eta r>0; \qquad b\equiv\eta\left(1+\frac{1}{\eta n-1}\right)>0.$$

Thus, in equilibrium, the unit rate of extraction chosen by all firms, ρ, is a linear, increasing function of β, the conjectural parameter. The faster the firms expect their rivals to extract the resource, the faster they will choose to extract the resource themselves. As will be shown, this self-fulfilling-prophecy aspect can generate instability in common-property markets.

How does the oligopolistic industry's extraction rate compare with the socially optimal extraction rate? The Hotelling rule states that, for Pareto-optimal allocation in the absence of extraction costs and uncertainty, the price of the resource should increase at the rate of interest; that is $\hat{P}=r$. With constant price elasticity of demand, this implies

$$\hat{S}=-\eta r. \tag{20}$$

The conjectural parameter β determines whether or not the market outcome is optimal. Suppose $\beta=0$. Then, from (20) and (15), the equilibrium rate of extraction is the socially optimal rate. The intuition is that, with this particular value of β, the optimizing firm behaves as if it had well-defined property rights. It believes its rivals maintain a given extraction path independently of its own actions. In effect there is a given quantity of the resource available for it to extract; there is no need to speed up its extraction process in order to prelude extraction by its rivals. The results of Bolle and Kemp and Long correspond to this case.

Suppose $\beta>0$; this means that the firm believes that, of every unit of the resource it leaves unextracted, part will be extracted by its rivals. Then, because $b>0$, the extraction function (19) shows that $\rho>\eta r$, and hence $\hat{S}<-\eta r$. There is over-extraction (as predicted by Khalatbari and Sinn for a special case in which β is a technologically-determined positive constant). The larger is the conjectural parameter β, the greater is the degree of over-extraction.

3. Rational-expectations equilibrium

In the previous section it was shown that, corresponding to the infinity of possible conjectures about rivals' reactions, there are infinitely many dynamic equilibria. In this section it is asked whether adopting a stronger equilibrium definition, requiring conjectures to be rational in a sense about to be made precise, reduces the number of possible equilibria.

In a conjectural equilibrium, conjectures are correct at a point, in that the actual rate at which any firm sees its rivals extracting the resource, $S(t)\rho(n-1)/n$, is the same as the rate it conjectured for them, $\alpha(t)+\beta S(t)$. A stronger notion of equilibrium requires that conjectures be correct not only at the equilibrium point but also for some range around it. Suppose the size of the resource stock changes by some small amount ΔS (perhaps because new information becomes available). Then, given the conjectures β, a new conjectural equilibrium will be established where each firm observes its rivals' rate of extraction to increase by

$\Delta S\rho(n-1)/n$. Hence $\rho(n-1)/n$ is the actual marginal rate of extraction on the part of i's rivals. Now define a rational-expectations equilibrium to be such that $\rho(n-1)/n = \beta$, or equivalently:

$$\rho = \frac{n}{(n-1)}\beta. \tag{21}$$

Thus, at a rational-expectations equilibrium the actual marginal rate of extraction is equal to the conjectural marginal rate of extraction. Eq. (21)

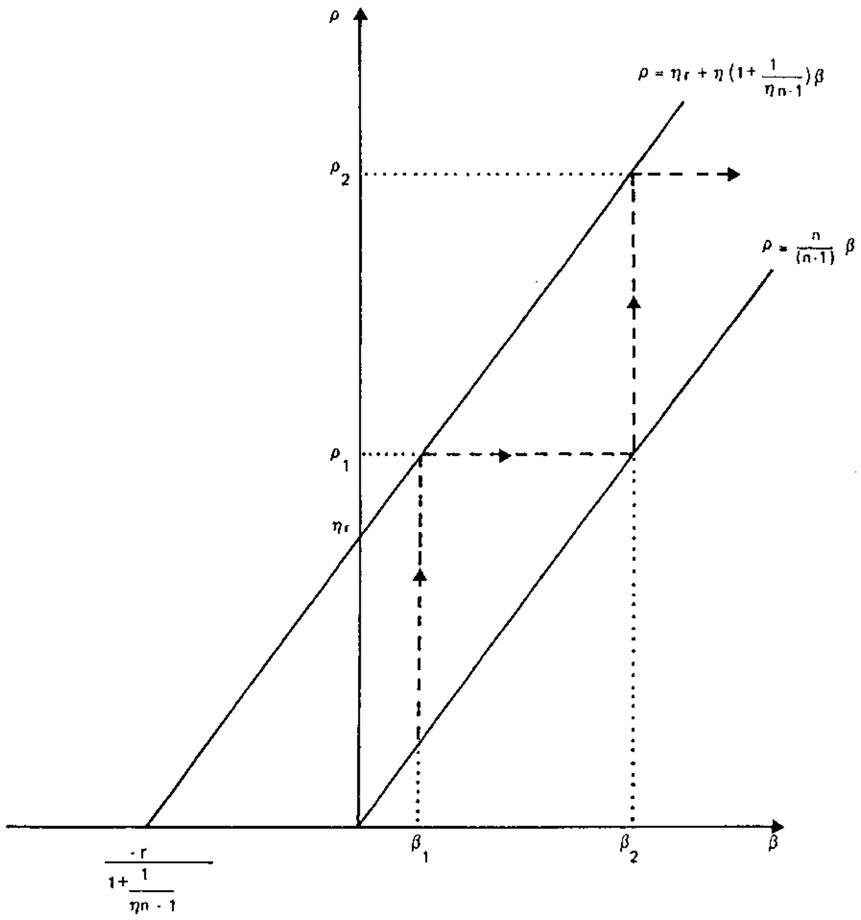

Fig. 11.1 The extraction function and the process of revising the conjectural parameter.

is represented in fig. 11. 1, which also shows the graph of the extraction function (19). If a rational-expectations equilibrium exists, it must be characterized by the point of intersection of the two lines. The location and the existence of this intersection point depend upon the elasticity of the market demand curve. Elementary manipulations show:

$$\eta \gtreqless 1 \Leftrightarrow b \gtreqless \frac{n}{n-1} \tag{22}$$

where $b \equiv \eta[1 + 1/(\eta n - 1)]$ is the slope of the extraction function as given by (19).

Consider first the case $\eta \geq 1$. Here, according to (22), the line depicting the extraction function (19) is at least as steep as the line given by (21). Since, from (19), the extraction function has a strictly positive intercept, this implies that the two lines cannot intersect for finite values of ρ and β in the range where $\rho > 0$, that is, where the total extraction per unit of stock is positive. The latter, however, is required by the transversality condition (9) in connection with eq. (15).

Suppose, instead, that $1/n < \eta < 1$. For this case, (22) clearly ensures that there is a point of intersection with $\rho > 0$. But again this point does not satisfy the transversality condition. To see this, note from (18b) that in the case of $\eta < 1$ the transversality condition requires $\beta < \eta r / \{[1-\eta][1 + 1/(n\eta - 1)]\}$. Eqs. (19) and (21), on the other hand, imply that the point of intersection is characterized by:

$$\beta = \frac{\eta r}{n/(n-1) - \eta(1 + 1/(n\eta - 1))}. \tag{23}$$

Elementary algebra shows that (18b) and (23) are incompatible.

Thus, any outcome in which the conjectural marginal extraction rate β and the actual marginal extraction rate $\rho(n-1)/n$ coincide does not satisfy the transversality condition of the individual firm. No rational-expectations equilibrium with finite extraction rates exists.

A rational-expectations equilibrium is a natural end-point for a disequilibrium adjustment process in which firms adjust their conjectures in the light of their observations of their rivals' actual behaviour. To demonstrate the implications of the non-existence of such an equilibrium, consider the firm's reactions to new information. If the system is in an equilibrium, then in the absence of exogenous disturbances the divergence between the actual marginal rate of extraction, $\rho(n-1)/n$, and the conjectural parameter β is not revealed. Suppose, however, there is new (public) information which causes the estimate of the size of the resource stock to be revised by some small amount. Now, given the conjectures, a new equilibrium path will be established and each firm will learn that[5] $\rho(n-1)/n > \beta$; i.e. that its conjecture about the rivals' marginal extraction rate was too conservative. This new information will cause it in some way to revise upwards its conjectural parameter β. According to the extraction function (19) a new equilibrium with a higher rate of extraction ρ per unit of stock is achieved. From (19) and (22):

$$\frac{\mathrm{d}}{\mathrm{d}\beta}\left[\frac{(n-1)}{n}\rho\right] = \frac{(n-1)}{n} b \gtreqless 1 \Leftrightarrow \eta \gtreqless 1. \tag{24}$$

This means that any change in the conjectural marginal rate of extraction translates into a larger, equal, or smaller change in the actual marginal rate of extraction as the absolute elasticity of demand is larger than, equal to, or smaller than unity, respectively. If $\eta < 1$, then, with a sequence of exogenous disturbances, both rates approach each other. However, as shown above, before they coincide the conjectural equilibrium ceases to exist. If $\eta \geq 1$, new information always causes there to be an equal or increased discrepancy between conjectured and actual marginal extraction rates; new information results in ever faster extraction.

In fig. 11. 1 the arrows depict this process for the particular case in which firms conjecture that their rivals' reactions to extra stock will be the same as their actual reaction at the last observation (with $\eta = 1$).

The non-existence of a rational-expectations equilibrium with finite extraction rates means that every possible equilibrium corresponding to conjectures of the form (1) is unstable in the sense that it is based on misapprehensions by firms about their rivals' behaviour: new information will cause firms to revise upwards their conjectures about their rivals' rates of extraction. This is true in particular of the equilibrium in which extraction occurs at the socially optimal rate (the $\beta = 0$ case).

4. Relationship to seepage models

The model developed above describes a common-property problem in which each firm has access to the whole pool of the resource. Comparisons were made with the results of the problem of Kemp and Long

[5]From the discussion in the preceding paragraphs, it is clear that the firm will never observe $\rho(n-1)/n < \beta$.

(1980), Khalatbari (1977), and Sinn (1983) in which the firms own separate oil wells between which there is seepage. It remains to show that the two problems are indeed comparable.

Suppose there are n symmetrically-placed oligopolists owning resource stocks of sizes $S_1, \ldots, S_n$, from which they extract at the rates $R_1, \ldots, R_n$. Let

$$S = \sum_{j=1}^{n} S_j, \qquad R = \sum_{j=1}^{n} R_j, \qquad S_{-i} = \sum_{\substack{j=1\\ j\neq i}}^{n} S_j \qquad R_{-i} = \sum_{\substack{j=1\\ j\neq i}}^{n} R_j.$$

Oil seeps between the ith well and the others[6] at a rate which is proportional to the difference between the size of the ith stock S_i and the average size of all the other stocks, $S_{-i}/(n-1)$. Then the single firm's decision problem can be formulated as:

$$\max_{R_i} \int_0^{\infty} P(R^{c}(t))R_i(t)\exp(-rt)\,\mathrm{d}t \tag{25}$$

subject to

$$\dot{S}_{-i}(t) = -R_i(t) + s\left(\frac{S_{-i}(t)}{n-1} - S_i(t)\right). \tag{26}$$

$$\dot{S}_i(t) = -R_{-i}(t) + s\left(S_i(t) - \frac{S_{-i}(t)}{n-1}\right), \tag{27}$$

$$R^{c}_{-i}(t) = \varepsilon(t) + \delta\left(S_i(t) - \frac{S_{-i}(t)}{n-1}\right) - \gamma S_{-i}(t). \tag{28}$$

where $S_i(0) = S_{-i}(0)/(n-1) = S_0/n > 0$, $R^{c}_{-i}(t)$, $R_i(t)$, $S_i(t)$, $S_{-i}(t) \geq 0$. Eqs. (26) and (27) describe the seepage law, where $s > 0$ is the seepage parameter. Eq. (28) expresses firm i's conjectural hypothesis about the extraction plans of its rivals: γ reflects firm i's conjecture that its rivals will extract at a rate dependent on the size of their stocks; and σ represents firm i's conjecture that its rivals will extract immediately a fraction δ/s of the net inflow of oil from the ith firm's holdings to its rivals' holdings. The model (25), (26), (27), and (28) reduces to the Kemp–Long model if $\delta = \gamma = 0$, to the model of Sinn if $\delta = s$, and to that

[6]It is not necessary to consider separately the other $(n-1)$ stocks because the ith firm's decision does not depend upon the way the resource is distributed among its rivals; it is thus sufficient to consider the aggregate variables S_{-i}, R_{-i}.

of Khalatbari[7] if $\delta = s$ and in addition $n \to \infty$.

To relate this seepage model to the model studied in this essay, first note that (26), (27), and (28) can be rewritten with S and S_{-i} as state variables, instead of S_i and S_{-i}, because $S_i = S - S_{-i}$:

$$\dot{S}(t) = -R(t), \tag{29}$$

$$\dot{S}_{-i}(t) = -R_{-i}(t) + s[S(t) - S_{-i}(t)n/(n-1)], \tag{30}$$

$$R_{-i}(t) = \varepsilon(t) + \delta S(t) + S_{-i}(t)(\gamma - \delta n/(n-1)). \tag{31}$$

Consider now the Kemp–Long case $\gamma = \delta = 0$. This is the same as problems (1), (2), and (3) with $\beta = 0$, except for the additional differential equation (30). However, the co-state variable of $S_{-i}(t)$ is zero since, given $S(t)$, a change is $S_{-i}(t)$ could not change the present value of firm i's profits. Hence, the marginal conditions for firm i's decision problem are the same, namely (5) and (7). Only if $S_{-i}(t) > S(t)$ (which would imply $S_i(t) < 0$) could $S_{-i}(t)$ affect firm i's decision problem: however, in a symmetric equilibrium, $S_i(t) = S_j(t)$, this possibility need not be considered. Thus, in equilibrium $S_{-i}(t)$ is an irrelevant state variable in firm i's decision problem and hence the Kemp–Long model is a special case of the model considered in sections 2 and 3.

For the model of Sinn (and of Khalatbari when $n \to \infty$), $\delta = s$. From (30) and (31), $\dot{S}_{-i}(t)$ and hence $S_{-i}(t)$ are independent of $S(t)$: the time path of the stocks of the resource under the properties of i's rivals is exogenous to firm i's decision problem. Hence, firm i conjectures its rivals' rates of extraction are $R^c_{-i}(t) = \alpha(t) + \beta S(t)$, where $\beta = \delta$ and $\alpha(t) = \varepsilon(t) + S_{-i}(t)(\gamma - \delta n/(n-1))$. For this case also, the seepage model is a special case of the model of sections 2 and 3.

A third situation in which the seepage model and the common-pool model coincide is when $\gamma = \delta n/(n-1)$. Then (31) reduces to (1) and again $S_{-i}(t)$ is an endogenous but irrelevant state variable in firm i's decision problem. In this case, firm i conjectures that its rivals react only to the size of the total resource stock and not to its distribution over the properties. Given this conjecture, firm i's own decision depends only on the total resource stock; the conjecture is self-confirming.

[7]Strictly speaking, this approach is not compatible with Khalatbari's model, since in that model it is implicitly assumed that firm i conjectures that the whole seepage inflow from the ith firm's holding to its rivals' holdings is immediately extracted by its rivals but not sold on the market (see Kemp and Long (1980, pp. 131–132)). This assumption is innocuous only in the limiting case of $n \to \infty$ (Sinn (1984)); henceforth we will interpret Khalatbari's result as describing the case of perfect competition.

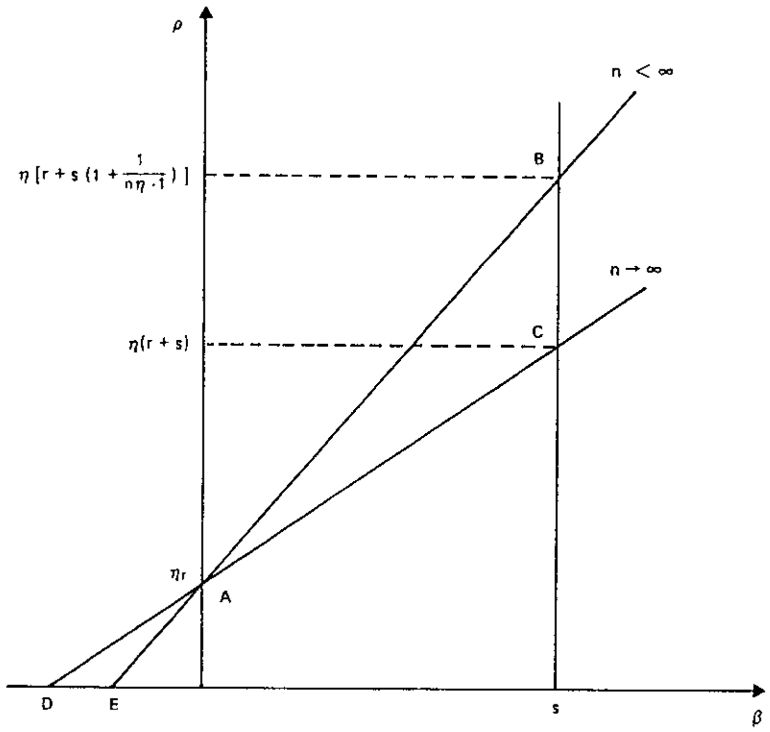

Figure 11.2 The relationship to seepage models.

Figure 11. 2 illustrates the possible equilibria for the seepage model. Possible equilibria lie along the line representing the extraction function (19): line EAB for the case of a finite number of firms and line DAG for the perfect-competition case. The outcomes described in the literature are special cases of this model: point A represents the Kemp–Long solution ($\delta = \beta = \gamma = 0$), point B the Sinn solution ($\delta = \beta = s$), and point C the Khalatbari solution ($\delta = \beta = s$, $n \to \infty$).

5. Concluding comments

When oligopolists exploiting a common-property resource have nontrivial conjectures about their rivals' actions, infinitely many equilibria are possible, including in particular the Cournot-type equilibria previously analysed in the literature. Conjectures have a self-confirming property: the faster a firm expects its rivals to extract the resource, the faster it will itself extract. There exists no equilibrium with finite extraction rates in which conjectures are locally correct. A consequence of this is that new information about the size of the resource stock will always cause the discrepancy between actual and conjectured reactions to widen; new information will upset any equilibrium and cause the speed of extraction to increase.[8]

The equilibrium concept of this essay was designed to be a direct generalization of the various equilibrium concepts already used in the literature on common-property resources (Bolle (1980), Dasgupta and Heal (1979), Kemp and Long (1980), Khalatbari (1977), and Sinn (1983)). The oligopoly problem described above is an example of a differential game. It therefore should be pointed out that the equilibrium concept used is not one of the concepts usually used in differential-game models; rather, it bears a closer resemblance to the notion of conjectural-variations equilibrium from static oligopoly theory. The closed-loop and open-loop solutions of differential games (see Starr and Ho (1969) for definitions) are both special cases of conjectural equilibria as defined in section 2 above. The open-loop equilibrium involves strategies which do not depend on the current size of the resource stock: it corresponds in this essay to the case $\beta=0$; i.e. the socially optimal equilibrium. A closed-loop equilibrium would occur in this model when the planned extraction rate is the same as the actual extraction rate at all time points and for all possible sizes of the stock. The rational-expectations equilibrium defined in section 3 is a local approximation to a closed-loop equilibrium; since no rational-expectations equilibrium exists, the analysis of section 3 constitutes a proof that there exists no closed-loop equilibrium in linear strategies with finite extraction rates. Closed-loop equilibria in common-property models have been examined by Reinganum and Stokey (1981) and Eswaren and Lewis (1982). Reinganum and Stokey showed that, for the continuous-time case with elastic demand, there exists a closed-loop equilibrium, involving immediate extraction of the entire stock of the resource. This is consistent with the results above. The proof of the non-existence of a rational-expectations equilibrium in section 3 necessarily assumes that β, and therefore ρ, are finite. Immediate extraction corresponds to infinite values of β and ρ. Clearly, this is consistent with

[8]This model, by assuming each firm sells the resource immediately it extracts it, ignores the possibility that the firm might stockpile the resource after extraction. On the importance of storage in exhaustible-resource models, see Hartwick (1981).

eqs. (19) and (21) being satisfied simultaneously; immediate extraction is therefore a rational-expectations equilibrium provided the transversality condition is satisfied. The analysis shows why elastic demand must be assumed in order to generate a closed-loop equilibrium: the transversality condition (18) is not satisfied when there is inelastic demand and infinite β. Furthermore, the adjustment dynamics sketched in section 3 provide some intuitive understanding of why the only closed-loop equilibrium involves immediate extraction.[9]

The model suggests that there is a presumption that a common-property resource will be inefficiently extracted and therefore that there is scope for government intervention. However, with finite extraction rates there are infinitely many equilibria – none a rational-expectations equilibrium – most resulting in over-extraction, but one resulting in socially optimal extraction. The size of the distortion is unpredictable; thus no rectifying system of taxes can be calculated. In contrast to economists' usual prescriptions, quantitative controls seem in this case to be superior to taxes and subsidies. For example, prorationing (fixing a maximum permissible rate of extraction by individual firms) or compulsory or voluntary unitization (operating the whole pool under a single decision-maker and then distributing the profits among the individual firms: that is, collusion among the firms) in effect make β zero and thus ensure that extraction takes place at the socially optimal rate. These are, in fact, methods used in regulating the petroleum industry: see Khoury (1969), McDonald (1971), chs. 9, 10), and Watkins (1970, 1977).

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[9]Closed-loop and open-loop equilibria in the common-property problem are further discussed in McMillan (1984).

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